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14. ABSTRACT
Linear dispersion relations for electrostatic waves in spatially inhomogeneous, current-carrying anisotropic plasma, where the equilibrium particle velocity distributions are modeled by various Lorentzian (kappa) distributions and by well-known bi-Maxwellian distribution, are presented. Spatial inhomogeneities, assumed to be weak, include density gradients, temperature gradients, and gradients (shear) in the parallel (to the ambient magnetic field) flow velocities associated with the current. In order to illustrate the distinguishing features of the kappa distributions, stability properties of the low frequency (lower than ion cyclotron frequency) and long perpendicular wavelength (longer than ion gyroradius) modes are studied in detail, and the results are contrasted with those for the bi-Maxwellian distribution. Specific attention is given to the drift waves, the current-driven ion-acoustic waves in the presence of velocity shear, the velocity shear-driven ion-acoustic modes, and the ion temperature-gradient driven modes. Growth rates of the drift wave instability and the current-driven ion-acoustic instability are reduced from their values for bi-Maxwellian distributions due to larger ion Landau damping rates associated with the kappa distributions. For the same reason, excitation conditions for these two instabilities are more stringent in the case of the kappa distributions. Growth rates of the velocity shear-driven ion-acoustic instability and the ion temperature-gradient driven instability are reduced from their values for bi-Maxwellian distribution as a consequence of the reduced adiabatic response of the electrons to the electrostatic potential perturbation. Frequencies of the drift waves and the ion-acoustic waves are also reduced in kappa-distribution plasmas due to the reduced adiabatic response of the electrons.

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Low frequency electrostatic waves in weakly inhomogeneous magnetoplasma modeled by Lorentzian (kappa) distributions

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Linear dispersion relations for electrostatic waves in spatially inhomogeneous, current-carrying anisotropic plasma, where the equilibrium particle velocity distributions are modeled by various Lorentzian (kappa) distributions and by well-known bi-Maxwellian distribution, are presented. Spatial inhomogeneities, assumed to be weak, include density gradients, temperature gradients, and gradients (shear) in the parallel (to the ambient magnetic field) flow velocities associated with the current. In order to illustrate the distinguishing features of the kappa distributions, stability properties of the low frequency (lower than ion cyclotron frequency) and long perpendicular wavelength (longer than ion gyroradius) modes are studied in detail, and the results are contrasted with those for the bi-Maxwellian distribution. Specific attention is given to the drift waves, the current-driven ion-acoustic waves in the presence of velocity shear, the velocity shear-driven ion-acoustic modes, and the ion temperature-gradient driven modes. Growth rates of the drift wave instability and the current-driven ion-acoustic instability are reduced from their values for bi-Maxwellian distribution due to larger ion Landau damping rates associated with the kappa distributions. For the same reason, excitation conditions for these two instabilities are more stringent in the case of the kappa distributions. Growth rates of the velocity shear-driven ion-acoustic instability and the ion temperature-gradient driven instability are reduced from their values for bi-Maxwellian distribution as a consequence of the reduced adiabatic response of the electrons to the electrostatic potential perturbation. Frequencies of the drift waves and the ion-acoustic waves are also reduced in kappa-distribution plasmas due to the reduced adiabatic response of the electrons.
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I. INTRODUCTION

In collisionless plasma, particle distribution in velocity space can depart considerably from a Maxwellian. For example, in naturally occurring plasmas, such as plasmas in the planetary magnetospheres and solar wind plasma, particle velocity distributions are observed to have a prominent non-Maxwellian (power-law) high-energy tail (for some references to observations, see Ref. 1). The appropriate distribution functions that can better model such particle distributions are the generalized Lorentzian distributions, also known as the kappa distributions.[2] The kappa distribution with a finite value of the spectral index κ has a power-law tail at velocities higher than the thermal velocity and approaches a Maxwellian distribution in the limit as $\kappa \to \infty$. Typically, space plasmas are observed to possess a spectral index κ in the range 2–6. The presence of a substantially larger number of suprathermal particles, which distinguishes kappa distribution from a Maxwellian, can significantly change the rate of resonant energy transfer between particles and plasma waves. Hence, it could change the growth or damping rate of the plasma waves, the excitation conditions for instability, as well as the rate of anomalous transport processes that rely on resonant wave-particle interaction. It is, therefore, interesting to study the stability properties of plasma waves when the equilibrium (unperturbed) state of the plasma is described by a kappa, rather than a Maxwellian, distribution.

In the last several years, plasma waves (electrostatic and electromagnetic) in homogeneous, unmagnetized and magnetized plasma have been studied by various authors[1,3–14] using different types of kappa distributions for the equilibrium state. In this paper, we concern ourselves with the electrostatic waves in spatially inhomogeneous, current-carrying anisotropic plasma, where the equilibrium particle velocity distributions are modeled by various kappa distributions. Spatial inhomogeneities, assumed to be weak, include density gradients, temperature gradients, and gradients (shear) in the parallel (to the ambient magnetic field) flow velocities associated with the current. Such equilibrium plasma configuration is representative of many space and laboratory plasmas. We consider three specific forms of the kappa distributions, namely, kappa-Maxwellian,[9] product bi-Lorentzian,[1] and bi-Lorentzian.[1] We first present the full dispersion relations for electrostatic waves, then present the versions of the dispersion relations that are more suitable for the study of low frequency (lower than ion cyclotron frequency) waves, and finally concentrate on the analysis of the low frequency and long perpendicular wavelength (longer than ion gyroradius) modes. In particular, the stability properties of drift waves, current-driven ion-acoustic waves in the presence of velocity shear, velocity shear-driven ion-acoustic modes, and ion temperature gradient driven modes are analyzed in detail. Such plasma modes are commonly observed in inhomogeneous plasma and are responsible for anomalous effects, such as diffusion, thermal conduction and resistivity. We also include in our presentation the corresponding dis-

persion relations for the bi-Maxwellian distribution and their analysis. The reasons for including the previously known results for the bi-Maxwellian distribution are: First, to check the correctness of the results for the kappa distributions by using the fact that the kappa distribution goes over to the Maxwellian distribution in the limit as $\kappa\to\infty$; second, to compare and to illustrate the distinguishing features of the kappa distributions.

The paper is organized in the following way: In Sec. II, we describe the general mathematical formalism for the derivation of dispersion relation in weakly inhomogeneous magnetoplasma. In Sec. III, we present the dispersion relations for the different equilibrium distribution functions described in Sec. II. Section IV is devoted to the analysis of the low frequency, long perpendicular wavelength modes for the different equilibrium distribution functions. The results are summarized in Sec. V.

II. GENERAL MATHEMATICAL FORMALISM

A. Equilibrium state (spatial inhomogeneity along the *x*-direction)

The dynamics of nonrelativistic, collisionless plasma is determined by the Vlasov equation

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla+\frac{q_\alpha}{m_\alpha}\left(\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\right]f_\alpha(\mathbf{r},\mathbf{v},t)=0, \qquad (1)$$

where f_α is the single-particle distribution function, q_α is the charge, and m_α is the mass of the plasma constituent α ($\alpha=e$ for the electrons and $\alpha=i$ for the ions), while $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic field, respectively. The distribution function $f_{\alpha0}$ for the steady equilibrium state of spatially inhomogeneous plasma immersed in a uniform magnetic field $\mathbf{B}_0$ obeys the time-independent Vlasov equation

$$\left(\mathbf{v}\cdot\nabla+\frac{q_\alpha}{m_\alpha c}\mathbf{v}\times\mathbf{B}_0\cdot\frac{\partial}{\partial\mathbf{v}}\right)f_{\alpha0}(\mathbf{r},\mathbf{v})=0 \qquad (2)$$

and it can be constructed from the constants of motion of the charged particles. If we adopt a cylindrical coordinate system in velocity space with its z-axis parallel to $\mathbf{B}_0$, so that $v_x=v_\perp\cos\varphi$, $v_y=v_\perp\sin\varphi$ and $v_z\equiv\mathbf{v}\cdot\mathbf{B}_0/B_0=v_\parallel$, then the constants of motion are $x+v_y/\Omega_\alpha$, $y-v_x/\Omega_\alpha$, $v_\perp^2(=v_x^2+v_y^2)$, and $v_\parallel$. Here, $\Omega_\alpha=q_\alpha B_0/(m_\alpha c)$ is the cyclotron frequency of the charged particle species α. We assume that the plasma is inhomogeneous only along the x direction. Then, the most general $f_{\alpha0}$ is a function of $\xi=x+v_y/\Omega_\alpha$, $v_\perp^2$, and $v_\parallel$. That $f_{\alpha0}=f_{\alpha0}(\xi,v_\perp^2,v_\parallel)$ is a solution of Eq. (2) can be verified by direct substitution. If the spatial gradients are weak, $f_{\alpha0}(\xi,v_\perp^2,v_\parallel)$ can be expanded in a Taylor series about $\xi=0$. Thus, retaining only the terms that are linear in the gradients, we have

$$f_{\alpha0}(\xi,v_\perp^2,v_\parallel)\equiv[1+\xi/L(v_\perp^2,v_\parallel)]F_{\alpha0}(v_\perp^2,v_\parallel), \qquad (3)$$

where $F_{\alpha0}(v_\perp^2,v_\parallel)\equiv f_{\alpha0}(\xi=0,v_\perp^2,v_\parallel)$ and $1/L(v_\perp^2,v_\parallel)\equiv[(\partial f_{\alpha0}/\partial\xi)/f_{\alpha0}]_{\xi=0}$, which contains the essential features of the inhomogeneous plasma including density gradient, temperature gradient, and shear in the flow velocity. In the following, we consider the different specific forms of $f_{\alpha0}(\xi,v_\perp^2,v_\parallel)$ mentioned in the Introduction, and derive the respective expressions for $1/L$.

(a) Bi-Maxwellian (BM)

$$f_{\alpha0}^{\mathrm{BM}}(\xi,v_\perp^2,v_\parallel)=\frac{n_{\alpha0}(\xi)}{\pi^{3/2}\theta_{\alpha\perp}^2(\xi)\theta_{\alpha\parallel}(\xi)}\times\exp\left[-\frac{v_\perp^2}{\theta_{\alpha\perp}^2(\xi)}-\frac{u_{\alpha\parallel}^2(\xi)}{\theta_{\alpha\parallel}^2(\xi)}\right], \qquad (4)$$

where $u_{\alpha\parallel}(\xi)\equiv v_\parallel-V_{\alpha0}(\xi)$ and $V_{\alpha0}(\xi)$ is the inhomogeneous parallel flow velocity. Then, according to Eq. (3), we find

$$\frac{1}{L}=\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}-\frac{2}{L_{\theta\alpha\perp}}\left(1-\frac{v_\perp^2}{\theta_{\alpha\perp}^2}\right)+2\left(\frac{1}{L_{\theta\alpha\parallel}}\frac{u_{\alpha\parallel}^2}{\theta_{\alpha\parallel}^2}+\frac{u_{\alpha\parallel}V'_{\alpha0}}{\theta_{\alpha\parallel}^2}\right) \qquad (5)$$

and

$$F_{\alpha0}^{\mathrm{BM}}(v_\perp^2,v_\parallel)=\frac{n_{\alpha0}}{\pi^{3/2}\theta_{\alpha\perp}^2\theta_{\alpha\parallel}}\exp\left(-\frac{v_\perp^2}{\theta_{\alpha\perp}^2}-\frac{u_{\alpha\parallel}^2}{\theta_{\alpha\parallel}^2}\right). \qquad (6)$$

Here $1/L_{n\alpha}=(d\ln n_{\alpha0}/d\xi)_{\xi=0}$, $1/L_{\theta\alpha\parallel}=(d\ln\theta_{\alpha\parallel}/d\xi)_{\xi=0}$, $1/L_{\theta\alpha\perp}=(d\ln\theta_{\alpha\perp}/d\xi)_{\xi=0}$, and $V'_{\alpha0}=(dV_{\alpha0}/d\xi)_{\xi=0}$. It is to be understood that $n_{\alpha0}$, $V_{\alpha0}$, $\theta_{\alpha\parallel}$, and $\theta_{\alpha\perp}$ appearing in Eqs. (5) and (6) denote their values at $\xi=0$. We have suppressed the arguments for simplicity in notations. The function $F_{\alpha0}^{\mathrm{BM}}(v_\perp^2,v_\parallel)$ is normalized to the density $n_{\alpha0}$, and the thermal speeds $\theta_{\alpha\parallel}$ and $\theta_{\alpha\perp}$ are related to the particle temperatures, $T_{\alpha\parallel}$ and $T_{\alpha\perp}$, by $\theta_{\alpha\parallel}^2=2T_{\alpha\parallel}/m_\alpha$, $\theta_{\alpha\perp}^2=2T_{\alpha\perp}/m_\alpha$, where the definitions of $T_{\alpha\parallel}$ and $T_{\alpha\perp}$ are

$$n_{\alpha0}T_{\alpha\parallel}=2\pi m_\alpha\int dv_\perp dv_\parallel v_\perp(v_\parallel-V_{\alpha0})^2F_{\alpha0}(v_\perp^2,v_\parallel),$$
$$n_{\alpha0}T_{\alpha\perp}=\pi m_\alpha\int dv_\perp dv_\parallel v_\perp^3F_{\alpha0}(v_\perp^2,v_\parallel). \qquad (7)$$

(b) Kappa-Maxwellian (KM)

$$f_{\alpha0}^{\mathrm{KM}}(\xi,v_\perp^2,v_\parallel)=\frac{n_{\alpha0}(\xi)}{\pi^{3/2}\theta_{\alpha\perp}^2(\xi)\theta_{\alpha\parallel}(\xi)}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\times\left[1+\frac{u_{\alpha\parallel}^2(\xi)}{\kappa\theta_{\alpha\parallel}^2(\xi)}\right]^{-\kappa}\exp\left[-\frac{v_\perp^2}{\theta_{\alpha\perp}^2(\xi)}\right], \qquad (8)$$

where Γ is the gamma function and $u_{\alpha\parallel}(\xi)\equiv v_\parallel-V_{\alpha0}(\xi)$. Then, according to Eq. (3), we find

$$\frac{1}{L}=\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}-\frac{2}{L_{\theta\alpha\perp}}\left(1-\frac{v_\perp^2}{\theta_{\alpha\perp}^2}\right)+2\left(\frac{1}{L_{\theta\alpha\parallel}}\frac{u_{\alpha\parallel}^2}{\theta_{\alpha\parallel}^2}+\frac{u_{\alpha\parallel}V'_{\alpha0}}{\theta_{\alpha\parallel}^2}\right)\left(1+\frac{u_{\alpha\parallel}^2}{\kappa\theta_{\alpha\parallel}^2}\right)^{-1} \qquad (9)$$

and

$$F_{\alpha0}^{\mathrm{KM}}(v_\perp^2,v_\parallel)=\frac{n_{\alpha0}}{\pi^{3/2}\theta_{\alpha\perp}^2\theta_{\alpha\parallel}}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\times\left(1+\frac{u_{\alpha\parallel}^2}{\kappa\theta_{\alpha\parallel}^2}\right)^{-\kappa}\exp\left(-\frac{v_\perp^2}{\theta_{\alpha\perp}^2}\right). \quad (10)$$

As before, $n_{\alpha0}$, $V_{\alpha0}$, $\theta_{\alpha\parallel}$, and $\theta_{\alpha\perp}$ in Eqs. (9) and (10) refer to their values at $\xi=0$. The function $F_{\alpha0}^{\mathrm{KM}}(v_\perp^2,v_\parallel)$ is normalized to the particle density $n_{\alpha0}$, while $\theta_{\alpha\parallel}$ and $\theta_{\alpha\perp}$ are related to the particle parallel and perpendicular temperatures, defined by Eq. (7) as $\theta_{\alpha\parallel}^2=[(2\kappa-3)/\kappa](T_{\alpha\parallel}/m_\alpha)$ and $\theta_{\alpha\perp}^2=2T_{\alpha\perp}/m_\alpha$ for $\kappa>3/2$.

(c) Product Bi-Lorentzian (PBL)

$$f_{\alpha0}^{\mathrm{PBL}}(\xi,v_\perp^2,v_\parallel)=\frac{n_{\alpha0}(\xi)}{\pi^{3/2}\theta_{\alpha\perp}^2(\xi)\theta_{\alpha\parallel}(\xi)}\frac{\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)}\times\left[1+\frac{u_{\alpha\parallel}^2(\xi)}{\kappa_\parallel\theta_{\alpha\parallel}^2(\xi)}\right]^{-(\kappa_\parallel+1)}\times\left[1+\frac{v_\perp^2}{\kappa_\perp\theta_{\alpha\perp}^2(\xi)}\right]^{-(\kappa_\perp+1)}, \quad (11)$$

where $u_{\alpha\parallel}(\xi)\equiv v_\parallel-V_{\alpha0}(\xi)$. Then, according to Eq. (3), we find

$$\frac{1}{L}=\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}-\frac{2}{L_{\theta\alpha\perp}}+\frac{2(\kappa_\perp+1)}{\kappa_\perp}\frac{1}{L_{\theta\alpha\perp}}\frac{v_\perp^2}{\theta_{\alpha\perp}^2}\times\left(1+\frac{v_\perp^2}{\kappa_\perp\theta_{\alpha\perp}^2}\right)^{-1}+\frac{2(\kappa_\parallel+1)}{\kappa_\parallel}\left(\frac{1}{L_{\theta\alpha\parallel}}\frac{u_{\alpha\parallel}^2}{\theta_{\alpha\parallel}^2}+\frac{u_{\alpha\parallel}V'_{\alpha0}}{\theta_{\alpha\parallel}^2}\right)\times\left(1+\frac{u_{\alpha\parallel}^2}{\kappa_\parallel\theta_{\alpha\parallel}^2}\right)^{-1} \quad (12)$$

and

$$F_{\alpha0}^{\mathrm{PBL}}(v_\perp^2,v_\parallel)=\frac{n_{\alpha0}}{\pi^{3/2}\theta_{\alpha\perp}^2\theta_{\alpha\parallel}}\frac{\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)}\times\left(1+\frac{u_{\alpha\parallel}^2}{\kappa_\parallel\theta_{\alpha\parallel}^2}\right)^{-(\kappa_\parallel+1)}\left(1+\frac{v_\perp^2}{\kappa_\perp\theta_{\alpha\perp}^2}\right)^{-(\kappa_\perp+1)}. \quad (13)$$

As before, $n_{\alpha0}$, $V_{\alpha0}$, $\theta_{\alpha\parallel}$, and $\theta_{\alpha\perp}$ in Eqs. (12) and (13) refer to their values at $\xi=0$. The function $F_{\alpha0}^{\mathrm{PBL}}(v_\perp^2,v_\parallel)$ is normalized to the particle density $n_{\alpha0}$, while $\theta_{\alpha\parallel}$ and $\theta_{\alpha\perp}$ are related to the particle parallel and perpendicular temperatures, defined by Eq. (7), as $\theta_{\alpha\parallel}^2=[(2\kappa_\parallel-1)/\kappa_\parallel](T_{\alpha\parallel}/m_\alpha)$ and $\theta_{\alpha\perp}^2=[(\kappa_\perp-1)/\kappa_\perp](2T_{\alpha\perp}/m_\alpha)$, for $\kappa_\parallel>1/2$ and $\kappa_\perp>1$. Here we have allowed the possibility of different values of the spectral index in the parallel and the perpendicular directions. If $\kappa_\parallel=\kappa_\perp=\kappa$, (13) corresponds to the product bi-Lorentzian listed by Summers and Thorne.[1]

(d) Bi-Lorentzian (BL)

$$f_{\alpha0}^{\mathrm{BL}}(\xi,v_\perp^2,v_\parallel)=\frac{n_{\alpha0}(\xi)}{\pi^{3/2}\theta_{\alpha\perp}^2(\xi)\theta_{\alpha\parallel}(\xi)}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\times\left[1+\frac{v_\perp^2}{\kappa\theta_{\alpha\perp}^2(\xi)}+\frac{u_{\alpha\parallel}^2(\xi)}{\kappa\theta_{\alpha\parallel}^2(\xi)}\right]^{-(\kappa+1)}, \quad (14)$$

where $u_{\alpha\parallel}(\xi)\equiv v_\parallel-V_{\alpha0}(\xi)$. Then, according to Eq. (3), we have

$$\frac{1}{L}=\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}-\frac{2}{L_{\theta\alpha\perp}}+\frac{2(\kappa+1)}{\kappa}\times\left(\frac{1}{L_{\theta\alpha\perp}}\frac{v_\perp^2}{\theta_{\alpha\perp}^2}+\frac{1}{L_{\theta\alpha\parallel}}\frac{u_{\alpha\parallel}^2}{\theta_{\alpha\parallel}^2}+\frac{u_{\alpha\parallel}V'_{\alpha0}}{\theta_{\alpha\parallel}^2}\right)\times\left(1+\frac{v_\perp^2}{\kappa\theta_{\alpha\perp}^2}+\frac{u_{\alpha\parallel}^2}{\kappa\theta_{\alpha\parallel}^2}\right)^{-1} \quad (15)$$

and

$$F_{\alpha0}^{\mathrm{BL}}(v_\perp^2,v_\parallel)=\frac{n_{\alpha0}}{\pi^{3/2}\theta_{\alpha\perp}^2\theta_{\alpha\parallel}}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\times\left(1+\frac{v_\perp^2}{\kappa\theta_{\alpha\perp}^2}+\frac{u_{\alpha\parallel}^2}{\kappa\theta_{\alpha\parallel}^2}\right)^{-(\kappa+1)}, \quad (16)$$

where $n_{\alpha0}$, $V_{\alpha0}$, $\theta_{\alpha\parallel}$, and $\theta_{\alpha\perp}$ in Eqs. (15) and (16) denote the values of these quantities at $\xi=0$. The function $F_{\alpha0}^{\mathrm{BL}}(v_\perp^2,v_\parallel)$ is normalized to particle density $n_{\alpha0}$, while $\theta_{\alpha\parallel}$ and $\theta_{\alpha\perp}$ are related to the particle parallel and perpendicular temperatures, defined by Eq. (7), as $\theta_{\alpha\parallel}^2=[(2\kappa-3)/\kappa](T_{\alpha\parallel}/m_\alpha)$ and $\theta_{\alpha\perp}^2=[(2\kappa-3)/\kappa](T_{\alpha\perp}/m_\alpha)$, for $\kappa>3/2$.

It may be verified that for all the choices of $f_{\alpha0}(\xi,v_\perp^2,v_\parallel)$ the equilibrium values of the density and the temperatures are: $n_\alpha(x)=n_{\alpha0}(1+x/L_{n\alpha})$, $T_{\alpha\parallel}(x)=T_{\alpha\parallel}(1+x/L_{T\alpha\parallel})$, and $T_{\alpha\perp}(x)=T_{\alpha\perp}(1+x/L_{T\alpha\perp})$ to the lowest order in (x/L), where $L_{T\alpha\parallel}=L_{\theta\alpha\parallel}/2$ and $L_{T\alpha\perp}=L_{\theta\alpha\perp}/2$ are the scale lengths of the temperature gradients. Charge neutrality of the equilibrium state requires $L_{ne}=L_{ni}$. There is an equilibrium current density, which is a combination of the diamagnetic current density and the current density due to flow velocity along the ambient magnetic field $\mathbf{B}_0$. The self-consistent magnetic field due to the equilibrium current density is assumed to be negligibly small compared to the main ambient magnetic field $\mathbf{B}_0$.

For simplicity of notations, we assumed above that the spectral index κ has the same value for both electron and ion distributions. However, the analysis in the following sections can be easily generalized to allow different values of κ for the two charged populations. In Fig. 1 we have presented $G(u)\equiv(2\pi)^{3/2}(V_{T\alpha\parallel}/n_{\alpha0})\int dv_\perp v_\perp F_{\alpha0}$ as a function of $u\equiv(v_\parallel-V_{\alpha0})/V_{T\alpha\parallel}$, where $V_{T\alpha\parallel}=(T_{\alpha\parallel}/m_\alpha)^{1/2}$, for the four model velocity distributions described above. It may be verified that $G(u)$ for bi-Lorentzian is same as that for kappa-Maxwellian. As is well known in plasma physics, the function $G(u)$ plays important role in determining the stability properties of plasma waves.

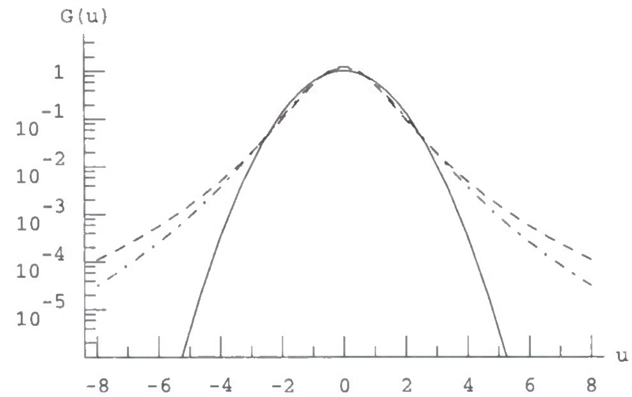

FIG. 1. Comparison of $G(u)\equiv(2\pi)^{3/2}(V_{T\alpha\parallel}/n_{\alpha0})\int dv_\perp v_\perp F_{\alpha0}$ vs $u\equiv(v_\parallel-V_{\alpha0})/V_{T\alpha\parallel}$. Solid curve is for bi-Maxwellian distribution, dashed curve is for kappa-Maxwellian ($\kappa=3$) distribution, and dashed-dotted curve is for product bi-Lorentzian ($\kappa_\parallel=3$) distribution. $G(u)$ for bi-Lorentzian is the same as that for kappa-Maxwellian.

B. Perturbed state (electrostatic perturbation) and dispersion relation

We now consider electrostatic perturbation so that the perturbed electric field $E_1(\mathbf{r},t)$ is given by $E_1(\mathbf{r},t)=-\nabla\phi_1(\mathbf{r},t)$, where $\phi_1(\mathbf{r},t)$ is the perturbed potential. The equations to be solved are the linearized Vlasov equation for the perturbed distribution function $f_{\alpha1}(\mathbf{r},\mathbf{v},t)$ and the Poisson equation for $\phi_1(\mathbf{r},t)$. They are

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla+\frac{q_\alpha}{m_\alpha c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial}{\partial\mathbf{v}}\right]f_{\alpha1}(\mathbf{r},\mathbf{v},t)-\frac{q_\alpha}{m_\alpha}\nabla\phi_1\cdot\frac{\partial}{\partial\mathbf{v}}f_{\alpha0}(\mathbf{r},\mathbf{v})=0 \quad (17)$$

and

$$\nabla^2\phi_1(\mathbf{r},t)=-4\pi\sum_\alpha q_\alpha\int d\mathbf{v}f_{\alpha1}(\mathbf{r},\mathbf{v},t). \quad (18)$$

In solving Eqs. (17) and (18) we adopt the "local approximation"[15] and assume that the perturbed quantities have space-time dependence of the form, $\mathbf{A}_1(\mathbf{r},t)=\tilde{\mathbf{A}}_1(\mathbf{k},\omega)\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, where $\mathbf{k}=(0,k_\perp,k_\parallel)$. The "local approximation" requires $k_\perp\gg\partial/\partial x\gg 1/L$ and $\rho_\alpha/L\ll1$, where ρ_α is the gyroradius of the particle α and L represents the typical scale length of spatial inhomogeneity. As is well known, the "local approximation" retains the leading-order effects of the spatial gradients. Using Eq. (3) for $f_{\alpha0}(\mathbf{r},\mathbf{v})$ and solving Eqs. (17) and (18) by the standard procedure,[16] the linear dispersion relation for electrostatic waves in inhomogeneous plasma, under the "local approximation," is obtained as

$$1+\sum_\alpha\frac{4\pi q_\alpha^2}{m_\alpha k^2}\sum_{n=-\infty}^{+\infty}\int d\mathbf{v}\frac{J_n^2(\mu_\alpha)}{\omega-k_\parallel v_\parallel+n\Omega_\alpha}\times\left(\frac{k_\perp}{\Omega_\alpha}\frac{1}{L}+k_\parallel\frac{\partial}{\partial v_\parallel}-2n\Omega_\alpha\frac{\partial}{\partial v_\perp^2}\right)F_{\alpha0}(v_\perp^2,v_\parallel)=0, \quad (19)$$

where $J_n(\mu_\alpha)$ is the Bessel function of the first kind and $\mu_\alpha=k_\perp v_\perp/\Omega_\alpha$.

In this paper we wish to study in detail the low frequency waves, $(\bar{\omega}_\alpha,k_\parallel V_{T\alpha\parallel})\ll\Omega_\alpha$,[16] in inhomogeneous plasma, where $\bar{\omega}_\alpha=\omega-k_\parallel V_{\alpha0}$. The reduced dispersion relation, which is adequate for the study of such low frequency waves, is obtained by first rewriting Eq. (19) as

$$1-\sum_\alpha\frac{4\pi q_\alpha^2}{m_\alpha k^2}\int d\mathbf{v}\left\{2\frac{\partial}{\partial v_\perp^2}-\sum_{n=-\infty}^{+\infty}\frac{J_n^2(\mu_\alpha)}{\omega-k_\parallel v_\parallel+n\Omega_\alpha}\times\left[\frac{k_\perp}{\Omega_\alpha}\frac{1}{L}+k_\parallel\frac{\partial}{\partial v_\parallel}+2(\omega-k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]\right\}F_{\alpha0}(v_\perp^2,v_\parallel)=0 \quad (20)$$

after using $n\Omega_\alpha=\omega-k_\parallel v_\parallel+n\Omega_\alpha-(\omega-k_\parallel v_\parallel)$ and the Bessel function identity $\sum_n J_n^2=1$, and then keeping only the $n=0$ term in the summation. The result is

$$1-\sum_\alpha\frac{4\pi q_\alpha^2}{m_\alpha k^2}\int d\mathbf{v}\left\{2[1-J_0^2(\mu_\alpha)]\frac{\partial}{\partial v_\perp^2}-\frac{J_0^2(\mu_\alpha)}{\omega-k_\parallel v_\parallel}\times\left(\frac{k_\perp}{\Omega_\alpha}\frac{1}{L}+k_\parallel\frac{\partial}{\partial v_\parallel}\right)\right\}F_{\alpha0}(v_\perp^2,v_\parallel)=0. \quad (21)$$

It can be verified from the results presented in Secs. III and IV that, under the conditions $(\bar{\omega}_\alpha,k_\parallel V_{T\alpha\parallel})\ll\Omega_\alpha$, the $n\neq0$ terms in the summation in Eq. (20) are negligibly small compared to the $n=0$ term, for all the equilibrium velocity distributions.

In the next section, we present the dispersion relations that are obtained for the different equilibrium distribution functions after substituting the specific expressions for $1/L$ and $F_{\alpha0}$ into Eqs. (19) and (21) and evaluating the velocity integrals.

III. DISPERSION RELATIONS FOR THE MODEL DISTRIBUTION FUNCTIONS

(a) Bi-Maxwellian (BM): When $F_{\alpha0}(v_\perp^2,v_\parallel)$ is a bi-Maxwellian, given by Eq. (6), and $1/L$ is given by Eq. (5), Eq. (19) yields the dispersion relation

$$1-\sum_\alpha\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\sum_{n=-\infty}^{+\infty}\Gamma_n(\beta_\alpha)\left\{\left[\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2\beta_\alpha}{L_{\theta\alpha\perp}}\frac{\Gamma_n'}{\Gamma_n}\right)+\frac{\theta_{\alpha\parallel}^2}{\theta_{\alpha\perp}^2}n\Omega_\alpha\right]\frac{Z^{\mathrm{BM}}(\xi_{n\alpha})}{k_\parallel\theta_{\alpha\parallel}}+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar{\omega}_\alpha+n\Omega_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\xi_{n\alpha}}Z^{\mathrm{BM}}(\xi_{n\alpha})\right\}=0, \quad (22)$$

where $\omega_{p\alpha}^2=4\pi q_\alpha^2 n_{\alpha0}/m_\alpha$, $\beta_\alpha=k_\perp^2\theta_{\alpha\perp}^2/(2\Omega_\alpha^2)$, $\bar{\omega}_\alpha=\omega-k_\parallel V_{\alpha0}$, $\xi_{n\alpha}=(\bar{\omega}_\alpha+n\Omega_\alpha)/(k_\parallel\theta_{\alpha\parallel})$, and Z^{BM} is the well-known plasma

dispersion function associated with the bi-Maxwellian parallel velocity distribution, defined by[17]

$$Z^{\mathrm{BM}}(\varsigma)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}ds\frac{\exp(-s^2)}{s-\varsigma},\quad \mathrm{Im}\,\varsigma>0 \tag{23}$$

and by the analytic continuation of Eq. (23) for $\mathrm{Im}\,\varsigma\leq 0$. Using $\theta_{\alpha\parallel}^2=2T_{\alpha\parallel}/m_\alpha$, $\theta_{\alpha\perp}^2=2T_{\alpha\perp}/m_\alpha$, $L_{\theta\alpha\parallel}=2L_{T\alpha\parallel}$, and $L_{\theta\alpha\perp}=2L_{T\alpha\perp}$, where $1/L_{T\alpha\parallel}=(d\ln T_{\alpha\parallel}/d\xi)_{\xi=0}$ and $1/L_{T\alpha\perp}=(d\ln T_{\alpha\perp}/d\xi)_{\xi=0}$, we rewrite Eq. (22) in the more familiar form,

$$1-\sum_\alpha\frac{1}{k^2\lambda_{D\alpha\parallel}^2}\sum_{n=-\infty}^{+\infty}\Gamma_n(b_\alpha)\left\{\left[\left(1-\frac{1}{2}\eta_{\alpha\parallel}+b_\alpha\eta_{\alpha\perp}\frac{\Gamma_n'}{\Gamma_n}\right)\omega_{*\alpha}+\frac{T_{\alpha\parallel}}{T_{\alpha\perp}}n\Omega_\alpha\right]\frac{Z^{\mathrm{BM}}(\xi_{n\alpha})}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}+\frac{1}{2}\left[1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}(\bar{\omega}_\alpha+n\Omega_\alpha)}{2k_\parallel^2V_{T\alpha\parallel}^2}\right]\frac{d}{d\xi_{n\alpha}}Z^{\mathrm{BM}}(\xi_{n\alpha})\right\}=0. \tag{24}$$

Here $\lambda_{D\alpha\parallel}^2=T_{\alpha\parallel}/(4\pi q_\alpha^2 n_{\alpha0})$, $\eta_{\alpha\parallel}=L_{n\alpha}/L_{T\alpha\parallel}$, $\eta_{\alpha\perp}=L_{n\alpha}/L_{T\alpha\perp}$, $\omega_{*\alpha}=k_\perp T_{\alpha\parallel}/(m_\alpha\Omega_\alpha L_{n\alpha})$, $\xi_{n\alpha}=(\bar{\omega}_\alpha+n\Omega_\alpha)/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$, $\Gamma_n(b_\alpha)=I_n(b_\alpha)\exp(-b_\alpha)$, where I_n is the modified Bessel function, and Γ_n' is the derivative of Γ_n with respect to its argument $b_\alpha=k_\perp^2T_{\alpha\perp}/(m_\alpha\Omega_\alpha^2)$.

The reduced dispersion relation for the low frequency waves in bi-Maxwellian plasma, derived from Eq. (21), is

$$1+\sum_\alpha\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\perp}^2}[1-\Gamma_0(\beta_\alpha)]-\sum_\alpha\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\Gamma_0(\beta_\alpha)\times\left\{\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2\beta_\alpha}{L_{\theta\alpha\perp}}\frac{\Gamma_0'}{\Gamma_0}\right)\frac{Z^{\mathrm{BM}}(\xi_\alpha)}{k_\parallel\theta_{\alpha\parallel}}+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar{\omega}_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\xi_\alpha}Z^{\mathrm{BM}}(\xi_\alpha)\right\}=0 \tag{25}$$

or, in terms of b_α, $\eta_{\alpha\parallel}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$, is

$$1+\sum_\alpha\frac{1-\Gamma_0(b_\alpha)}{k^2\lambda_{D\alpha\perp}^2}-\sum_\alpha\frac{\Gamma_0(b_\alpha)}{k^2\lambda_{D\alpha\parallel}^2}\times\left[\left(1-\frac{1}{2}\eta_{\alpha\parallel}+b_\alpha\eta_{\alpha\perp}\frac{\Gamma_0'}{\Gamma_0}\right)\frac{\omega_{*\alpha}}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}Z^{\mathrm{BM}}(\xi_\alpha)+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}\bar{\omega}_\alpha}{2k_\parallel^2V_{T\alpha\parallel}^2}\right)\frac{d}{d\xi_\alpha}Z^{\mathrm{BM}}(\xi_\alpha)\right]=0. \tag{26}$$

Here, $\xi_\alpha=\bar{\omega}_\alpha/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$.

(b) Kappa-Maxwellian (KM): When $F_{\alpha0}(v_\perp^2,v_\parallel)$ is a kappa-Maxwellian, given by Eq. (10), and $1/L$ is given by Eq. (9), Eq. (19) yields the dispersion relation

$$1-\sum_\alpha\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\sum_{n=-\infty}^{+\infty}\Gamma_n(\beta_\alpha)\times\left\{\left[\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2\beta_\alpha}{L_{\theta\alpha\perp}}\frac{\Gamma_n'}{\Gamma_n}\right)+\frac{\theta_{\alpha\parallel}^2}{\theta_{\alpha\perp}^2}n\Omega_\alpha\right]\frac{Z_\kappa^{\mathrm{KM}}(\varsigma_{n\alpha}^\kappa)}{k_\parallel\theta_{\alpha\parallel}}+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar{\omega}_\alpha+n\Omega_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\varsigma_{n\alpha}^\kappa}Z_\kappa^{\mathrm{KM}}(\varsigma_{n\alpha}^\kappa)\right\}=0, \tag{27}$$

where $\beta_\alpha=k_\perp^2\theta_{\alpha\perp}^2/(2\Omega_\alpha^2)$, $\varsigma_{n\alpha}^\kappa=(\bar{\omega}_\alpha+n\Omega_\alpha)/(k_\parallel\theta_{\alpha\parallel})$, and Z_κ^{KM} is the plasma dispersion function for the kappa-Maxwellian distribution, defined by

$$Z_\kappa^{\mathrm{KM}}(\varsigma)=\frac{\Gamma(\kappa)}{\pi^{1/2}\kappa^{1/2}\Gamma(\kappa-1/2)}\times\int_{-\infty}^{+\infty}\frac{ds}{(s-\varsigma)(1+s^2/\kappa)^\kappa},\quad \mathrm{Im}\,\varsigma>0 \tag{28}$$

and by the analytic continuation of Eq. (28) for $\mathrm{Im}\,\varsigma\leq 0$. It was first introduced and studied by Hellberg and Mace.[9] Using $\theta_{\alpha\parallel}^2=[(2\kappa-3)/\kappa](T_{\alpha\parallel}/m_\alpha)$, $\theta_{\alpha\perp}^2=2T_{\alpha\perp}/m_\alpha$, $L_{\theta\alpha\parallel}=2L_{T\alpha\parallel}$, and $L_{\theta\alpha\perp}=2L_{T\alpha\perp}$, we rewrite Eq. (27) in terms of b_α, $\eta_{\alpha\parallel}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$ as

$$1-\sum_\alpha\frac{1}{k^2\lambda_{D\alpha\parallel}^2}\frac{2\kappa}{2\kappa-3}\sum_{n=-\infty}^{+\infty}\Gamma_n(b_\alpha)\left\{\left(\frac{2\kappa-3}{2\kappa}\right)^{1/2}\times\left[\left(1-\frac{1}{2}\eta_{\alpha\parallel}+b_\alpha\eta_{\alpha\perp}\frac{\Gamma_n'}{\Gamma_n}\right)\omega_{*\alpha}+\frac{T_{\alpha\parallel}}{T_{\alpha\perp}}n\Omega_\alpha\right]\times\frac{Z_\kappa^{\mathrm{KM}}(\varsigma_{n\alpha}^\kappa)}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}+\frac{1}{2}\left[1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}(\bar{\omega}_\alpha+n\Omega_\alpha)}{2k_\parallel^2V_{T\alpha\parallel}^2}\right]\frac{d}{d\varsigma_{n\alpha}^\kappa}Z_\kappa^{\mathrm{KM}}(\varsigma_{n\alpha}^\kappa)\right\}=0 \tag{29}$$

where now $\varsigma_{n\alpha}^\kappa=[2\kappa/(2\kappa-3)]^{1/2}(\bar{\omega}_\alpha+n\Omega_\alpha)/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$.

The reduced dispersion relation for the low frequency waves in kappa-Maxwellian plasma, derived from Eq. (21), is

$$1+\sum_\alpha \frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\perp}^2}[1-\Gamma_0(\beta_\alpha)]-\sum_\alpha \frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\Gamma_0(\beta_\alpha) \times\left\{\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2\beta_\alpha}{L_{\theta\alpha\perp}}\frac{\Gamma_0'}{\Gamma_0}\right)\frac{Z_\kappa^{\mathrm{KM}}(\varsigma_\alpha^\kappa)}{k_\parallel\theta_{\alpha\parallel}} +\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar\omega_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\varsigma_\alpha^\kappa}Z_\kappa^{\mathrm{KM}}(\varsigma_\alpha^\kappa)\right\}=0 \tag{30}$$

or, in terms of b_α, $\eta_{\alpha\parallel}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$, is

$$1+\sum_\alpha \frac{1-\Gamma_0(b_\alpha)}{k^2\lambda_{D\alpha\perp}^2}-\sum_\alpha \frac{\Gamma_0(b_\alpha)}{k^2\lambda_{D\alpha\parallel}^2}\frac{2\kappa}{2\kappa-3}\left[\left(\frac{2\kappa-3}{2\kappa}\right)^{1/2} \times\left(1-\frac{1}{2}\eta_{\alpha\parallel}+b_\alpha\eta_{\alpha\perp}\frac{\Gamma_0'}{\Gamma_0}\right)\frac{\omega_{*\alpha}}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}Z_\kappa^{\mathrm{KM}}(\varsigma_\alpha^\kappa) +\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}\bar\omega_\alpha}{2k_\parallel^2V_{T\alpha\parallel}^2}\right)\frac{d}{d\varsigma_\alpha^\kappa}Z_\kappa^{\mathrm{KM}}(\varsigma_\alpha^\kappa)\right]=0. \tag{31}$$

Here, $\varsigma_\alpha^\kappa=[2\kappa/(2\kappa-3)]^{1/2}\bar\omega_\alpha/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$. In the limit as $\kappa\to\infty$, $\varsigma_{n\alpha}^\kappa\to\xi_{n\alpha}$, $\varsigma_\alpha^\kappa\to\xi_\alpha$, $Z_\kappa^{\mathrm{KM}}\to Z^{\mathrm{BM}}$, and thus Eqs. (29) and (31) become identical to their bi-Maxwellian counterparts Eqs. (24) and (26), respectively.

(c) Product Bi-Lorentzian (PBL): When $F_{\alpha0}(v_\perp^2,v_\parallel)$ is a product bi-Lorentzian, given by Eq. (13), and $1/L$ is given by Eq. (12), we use the series expansion of J_n^2 given by[18]

$$J_n^2(\mu_\alpha)=\sum_{p=|n|}^{+\infty}\frac{(-1)^{p-|n|}}{(p-|n|)!\,(p!)^2}\frac{\Gamma(2p+1)}{\Gamma(p+|n|+1)}\left(\frac{\mu_\alpha}{2}\right)^{2p} \tag{32}$$

in Eq. (19) and, after carrying out the integrations in $v_\parallel-v_\perp$ space, obtain the dispersion relation

$$1-\sum_\alpha \frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\sum_{n=-\infty}^{+\infty}\sum_{p=|n|}^{+\infty}g_{|n|,p}\frac{\kappa_\perp^p\Gamma(\kappa_\perp-p)}{\Gamma(\kappa_\perp)}\left(\frac{k_\perp^2\theta_{\alpha\perp}^2}{4\Omega_\alpha^2}\right)^p \times\left\{\left[\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2p}{L_{\theta\alpha\perp}}\right) +\frac{\theta_{\alpha\parallel}^2}{\theta_{\alpha\perp}^2}\left(\frac{\kappa_\perp-p}{\kappa_\perp}\right)n\Omega_\alpha\right]\frac{Z_\kappa^{\mathrm{PBL}}(\varsigma_{n\alpha}^\kappa)}{k_\parallel\theta_{\alpha\parallel}}+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha} -\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar\omega_\alpha+n\Omega_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\varsigma_{n\alpha}^\kappa}Z_\kappa^{\mathrm{PBL}}(\varsigma_{n\alpha}^\kappa)\right\}=0 \tag{33}$$

for $\kappa_\perp>p$. Here $\varsigma_{n\alpha}^\kappa=(\bar\omega_\alpha+n\Omega_\alpha)/(k_\parallel\theta_{\alpha\parallel})$,

$$g_{|n|,p}=\frac{(-1)^{p-|n|}}{p!\,(p-|n|)!}\frac{\Gamma(2p+1)}{\Gamma(p+|n|+1)}, \tag{34}$$

and Z_κ^{PBL} is the plasma dispersion function for the product bi-Lorentzian distribution, defined by

$$Z_\kappa^{\mathrm{PBL}}(\varsigma)=\frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)}\int_{-\infty}^{+\infty}\frac{ds}{(s-\varsigma)(1+s^2/\kappa_\parallel)^{\kappa_\parallel+1}},\quad \mathrm{Im}\,\varsigma>0 \tag{35}$$

and by the analytic continuation of Eq. (35) for $\mathrm{Im}\,\varsigma\le 0$. Apart from the multiplication factor, this function is the same as the dispersion function introduced and discussed by Summers and Thorne.[1] Using $\theta_{\alpha\parallel}^2=[(2\kappa_\parallel-1)/\kappa_\parallel](T_{\alpha\parallel}/m_\alpha)$, $\theta_{\alpha\perp}^2=[(\kappa_\perp-1)/\kappa_\perp](2T_{\alpha\perp}/m_\alpha)$, $L_{\theta\alpha\parallel}=2L_{T\alpha\parallel}$, and $L_{\theta\alpha\perp}=2L_{T\alpha\perp}$ we rewrite Eq. (33) in terms of b_α, $\eta_{\alpha\parallel}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$ as

$$1-\sum_\alpha \frac{1}{k^2\lambda_{D\alpha\parallel}^2}\frac{2\kappa_\parallel}{2\kappa_\parallel-1}\sum_{n=-\infty}^{+\infty}\sum_{p=|n|}^{+\infty}g_{|n|,p}\frac{\kappa_\perp^p\Gamma(\kappa_\perp-p)}{\Gamma(\kappa_\perp)}\left(\frac{\kappa_\perp-1}{2\kappa_\perp}b_\alpha\right)^p\left\{\left(\frac{2\kappa_\parallel-1}{2\kappa_\parallel}\right)^{1/2}\left[\left(1-\frac{1}{2}\eta_{\alpha\parallel}+p\eta_{\alpha\perp}\right)\omega_{*\alpha} +\frac{T_{\alpha\parallel}}{T_{\alpha\perp}}\left(\frac{\kappa_\perp}{\kappa_\perp-1}\right)n\Omega_\alpha\right]\frac{Z_\kappa^{\mathrm{PBL}}(\varsigma_{n\alpha}^\kappa)}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}+\frac{1}{2}\left[1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}(\bar\omega_\alpha+n\Omega_\alpha)}{2k_\parallel^2V_{T\alpha\parallel}^2}\right]\frac{d}{d\varsigma_{n\alpha}^\kappa}Z_\kappa^{\mathrm{PBL}}(\varsigma_{n\alpha}^\kappa)\right\}=0, \tag{36}$$

where now $\varsigma_{n\alpha}^\kappa=[2\kappa_\parallel/(2\kappa_\parallel-1)]^{1/2}(\bar\omega_\alpha+n\Omega_\alpha)/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$.

The reduced dispersion relation for the low frequency waves in product bi-Lorentzian plasma, derived from Eq. (21), is

$$1-\sum_\alpha \frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\perp}^2}\sum_{p=1}^{+\infty}g_{0,p}\frac{\kappa_\perp^{p-1}\Gamma(\kappa_\perp-p+1)}{\Gamma(\kappa_\perp)}\left(\frac{k_\perp^2\theta_{\alpha\perp}^2}{4\Omega_\alpha^2}\right)^p-\sum_\alpha \frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\sum_{p=0}^{+\infty}g_{0,p}\frac{\kappa_\perp^p\Gamma(\kappa_\perp-p)}{\Gamma(\kappa_\perp)}\left(\frac{k_\perp^2\theta_{\alpha\perp}^2}{4\Omega_\alpha^2}\right)^p \times\left\{\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2p}{L_{\theta\alpha\perp}}\right)\frac{Z_\kappa^{\mathrm{PBL}}(\varsigma_\alpha^\kappa)}{k_\parallel\theta_{\alpha\parallel}}+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{\alpha0}'}{\Omega_\alpha}-\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar\omega_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\varsigma_\alpha^\kappa}Z_\kappa^{\mathrm{PBL}}(\varsigma_\alpha^\kappa)\right\}=0 \tag{37}$$

or, in terms of b_α, $\eta_{\alpha\parallel}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$, is

$$1-\sum_{\alpha}\frac{1}{k^2\lambda_{D\alpha\perp}^2}\sum_{p=1}^{+\infty}g_{0,p}\frac{\kappa_\perp^p\Gamma(\kappa_\perp-p+1)}{(\kappa_\perp-1)\Gamma(\kappa_\perp)}\left(\frac{\kappa_\perp-1}{2\kappa_\perp}b_\alpha\right)^p-\sum_{\alpha}\frac{1}{k^2\lambda_{D\alpha\|}^2}\frac{2\kappa_\|}{2\kappa_\|-1}\sum_{p=0}^{+\infty}g_{0,p}\frac{\kappa_\perp^p\Gamma(\kappa_\perp-p)}{\Gamma(\kappa_\perp)}\left(\frac{\kappa_\perp-1}{2\kappa_\perp}b_\alpha\right)^p$$
$$\times\left[\left(\frac{2\kappa_\|-1}{2\kappa_\|}\right)^{1/2}\left(1-\frac{1}{2}\eta_{\alpha\|}+p\eta_{\alpha\perp}\right)\frac{\omega_{*\alpha}}{\sqrt{2}k_\|V_{T\alpha\|}}Z_\kappa^{\mathrm{PBL}}(\varsigma_\alpha^\kappa)+\frac{1}{2}\left(1-\frac{k_\perp}{k_\|}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\|}\frac{\omega_{*\alpha}\bar{\omega}_\alpha}{2k_\|^2V_{T\alpha\|}^2}\right)\frac{d}{d\varsigma_\alpha^\kappa}Z_\kappa^{\mathrm{PBL}}(\varsigma_\alpha^\kappa)\right]=0 \tag{38}$$

for $\kappa_\perp>p$. Here $\varsigma_\alpha^\kappa=[2\kappa_\|/(2\kappa_\|-1)]^{1/2}\bar{\omega}_\alpha/(\sqrt{2}k_\|V_{T\alpha\|})$. The condition $\kappa_\perp>p$, which arises from the integral $\int_0^\infty dv_\perp v_\perp^{2p+1}[1+v_\perp^2/(\kappa_\perp\theta_{\alpha\perp}^2)]^{-(\kappa_\perp+1)}$, does not present any difficulty in using Eqs. (36) and (38) for the study of long perpendicular wavelength ($b_\alpha\ll1$) modes, including finite-b_α corrections, if $\kappa_\perp\geq2$.

At this point, let us verify that Eq. (36) → Eq. (24) and Eq. (38) → Eq. (26), in the limit as $\kappa_\|,\kappa_\perp\to\infty$. To do this, we note that $\varsigma_{n\alpha}^\kappa\to\xi_{n\alpha}$, $Z_\kappa^{\mathrm{PBL}}\to Z^{\mathrm{BM}}$ in the limit as $\kappa_\|\to\infty$, and that $\kappa_\perp^p\Gamma(\kappa_\perp-p)/\Gamma(\kappa_\perp)\to1$ in the limit as $\kappa_\perp\to\infty$. As a result, Eq. (36) becomes

$$1-\sum_{\alpha}\frac{1}{k^2\lambda_{D\alpha\|}^2}\sum_{n=-\infty}^{+\infty}\sum_{p=|n|}^{+\infty}g_{|n|,p}\left(\frac{b_\alpha}{2}\right)^p$$
$$\times\left\{\left[\left(1-\frac{1}{2}\eta_{\alpha\|}+p\eta_{\alpha\perp}\right)\omega_{*\alpha}+\frac{T_{\alpha\|}}{T_{\alpha\perp}}n\Omega_\alpha\right]\frac{Z^{\mathrm{BM}}(\xi_{n\alpha})}{\sqrt{2}k_\|V_{T\alpha\|}}\right.$$
$$+\frac{1}{2}\left[1-\frac{k_\perp}{k_\|}\frac{V_{\alpha0}'}{\Omega_\alpha}-\eta_{\alpha\|}\frac{\omega_{*\alpha}(\bar{\omega}_\alpha+n\Omega_\alpha)}{2k_\|^2V_{T\alpha\|}^2}\right]$$
$$\left.\times\frac{d}{d\xi_{n\alpha}}Z^{\mathrm{BM}}(\xi_{n\alpha})\right\}=0. \tag{39}$$

Next, multiplying Eq. (32) by $v_\perp\exp[-m_\alpha v_\perp^2/(2T_{\alpha\perp})]$ and integrating from $v_\perp=0$ to $v_\perp=\infty$, we find

$$\sum_{p=|n|}^{+\infty}g_{|n|,p}\left(\frac{b_\alpha}{2}\right)^p=I_n(b_\alpha)\exp(-b_\alpha)\equiv\Gamma_n(b_\alpha) \tag{40}$$

and, hence,

$$\sum_{p=|n|}^{+\infty}pg_{|n|,p}\left(\frac{b_\alpha}{2}\right)^p=b_\alpha\frac{d}{db_\alpha}\Gamma_n(b_\alpha)\equiv b_\alpha\Gamma_n'(b_\alpha). \tag{41}$$

It is then evident that Eq. (39) is identical to Eq. (24) by virtue of Eqs. (40) and (41). In a similar manner, it can be verified that Eq. (38) → Eq. (26), in the limit as $\kappa_\|,\kappa_\perp\to\infty$.

(d) Bi-Lorentzian (BL): When $F_{\alpha0}(v_\perp^2,v_\|)$ is a bi-Lorentzian, given by Eq. (16), and $1/L$ is given by Eq. (15), we once again use the series expansion for J_n^2, given by Eq. (32), in Eq. (19) and after carrying out the integrations in $v_\|-v_\perp$ space obtain the dispersion relation

$$1-\sum_{\alpha}\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\|}^2}\sum_{n=-\infty}^{+\infty}\sum_{p=|n|}^{+\infty}g_{|n|,p}\frac{\kappa^p\Gamma(\kappa-p)}{\Gamma(\kappa)}\left(\frac{k_\perp^2\theta_{\alpha\perp}^2}{4\Omega_\alpha^2}\right)^p$$
$$\times\left\{\frac{k_\perp\theta_{\alpha\|}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\|}}+\frac{2p}{L_{\theta\alpha\perp}}\right)\frac{1}{k_\|\theta_{\alpha\|}}Z_{\kappa,p}^{\mathrm{BL}}(\varsigma_{n\alpha}^\kappa)\right.$$
$$+\frac{\theta_{\alpha\|}^2}{\theta_{\alpha\perp}^2}\left(\frac{\kappa-p}{\kappa}\right)\frac{n\Omega_\alpha}{k_\|\theta_{\alpha\|}}Z_{\kappa,p-1}^{\mathrm{BL}}(\varsigma_{n\alpha}^\kappa)+\frac{1}{2}\left(1-\frac{k_\perp}{k_\|}\frac{V_{\alpha0}'}{\Omega_\alpha}\right.$$
$$\left.\left.-\frac{k_\perp\theta_{\alpha\|}^2}{\Omega_\alpha L_{\theta\alpha\|}}\frac{\bar{\omega}_\alpha+n\Omega_\alpha}{k_\|^2\theta_{\alpha\|}^2}\right)\frac{d}{d\varsigma_{n\alpha}^\kappa}Z_{\kappa,p}^{\mathrm{BL}}(\varsigma_{n\alpha}^\kappa)\right\}=0 \tag{42}$$

for $\kappa>p$. Here $\varsigma_{n\alpha}^\kappa=(\bar{\omega}_\alpha+n\Omega_\alpha)/(k_\|\theta_{\alpha\|})$ and we have introduced the generalized dispersion function $Z_{\kappa,q}^{\mathrm{BL}}$, with $q=0,1,2,3,\ldots$, for the bi-Lorentzian distribution, defined by

$$Z_{\kappa,q}^{\mathrm{BL}}(\varsigma)\equiv\frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\int_{-\infty}^{+\infty}\frac{ds}{(s-\varsigma)(1+s^2/\kappa)^{\kappa-q}},$$
$$\mathrm{Im}\,\varsigma>0 \tag{43}$$

and by its analytic continuation for $\mathrm{Im}\,\varsigma\leq0$. We note that $Z_{\kappa,q=0}^{\mathrm{BL}}$ is identical to the dispersion function Z_κ^{KM} defined by Eq. (28). This has to do with the fact that $\int dv_\perp v_\perp F_{\alpha0}^{\mathrm{KM}}=\int dv_\perp v_\perp F_{\alpha0}^{\mathrm{BL}}$. Differentiating Eq. (43) with respect to ς and then performing integration by parts we find the relation

$$\frac{d}{d\varsigma}Z_{\kappa,q}^{\mathrm{BL}}(\varsigma)=-\frac{2(\kappa-q)}{\kappa}\left[\frac{\Gamma(\kappa)\Gamma(\kappa-q+1/2)}{\Gamma(\kappa-1/2)\Gamma(\kappa-q+1)}\right.$$
$$\left.+\varsigma Z_{\kappa,q-1}^{\mathrm{BL}}(\varsigma)\right] \tag{44}$$

which can be used to generate $Z_{\kappa,q}^{\mathrm{BL}}$ for $q=1,2,3,\ldots$ from $Z_{\kappa,q=0}^{\mathrm{BL}}$. As in the previous cases, we use $\theta_{\alpha\|}^2=[(2\kappa-3)/\kappa]\times(T_{\alpha\|}/m_\alpha)$, $\theta_{\alpha\perp}^2=[(2\kappa-3)/\kappa](T_{\alpha\perp}/m_\alpha)$, $L_{\theta\alpha\|}=2L_{T\alpha\|}$, and $L_{\theta\alpha\perp}=2L_{T\alpha\perp}$ to express Eq. (42) in terms of b_α, $\eta_{\alpha\|}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$. We obtain

$$1-\sum_{\alpha}\frac{1}{k^2\lambda_{D\alpha\parallel}^2}\frac{2\kappa}{2\kappa-3}\sum_{n=-\infty}^{+\infty}\sum_{p=|n|}^{+\infty}g_{|n|,p}\frac{\kappa^p\Gamma(\kappa-p)}{\Gamma(\kappa)}\left(\frac{2\kappa-3}{4\kappa}b_\alpha\right)^p\left\{\left[\left(1-\frac{1}{2}\eta_{\alpha\parallel}+p\eta_{\alpha\perp}\right)\frac{\omega_{*\alpha}}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}Z^{\mathrm{BL}}_{\kappa,p}(\varsigma^\kappa_{n\alpha})\right.\right.$$
$$\left.\left.+\frac{2(\kappa-p)}{2\kappa-3}\frac{T_{\alpha\parallel}}{T_{\alpha\perp}}\frac{n\Omega_\alpha}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}Z^{\mathrm{BL}}_{\kappa,p-1}(\varsigma^\kappa_{n\alpha})\right]\left(\frac{2\kappa-3}{2\kappa}\right)^{1/2}+\frac{1}{2}\left[1-\frac{k_\perp}{k_\parallel}\frac{V'_{\alpha0}}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}(\bar{\omega}_\alpha+n\Omega_\alpha)}{2k_\parallel^2V_{T\alpha\parallel}^2}\right]\frac{d}{d\varsigma^\kappa_{n\alpha}}Z^{\mathrm{BL}}_{\kappa,p}(\varsigma^\kappa_{n\alpha})\right\}=0, \qquad (45)$$

where now $\varsigma^\kappa_{n\alpha}=[2\kappa/(2\kappa-3)]^{1/2}(\bar{\omega}_\alpha+n\Omega_\alpha)/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$.

The reduced dispersion relation for the low frequency waves in bi-Lorentzian plasma, derived from Eq. (21), is

$$1-\sum_\alpha\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\perp}^2}\sum_{p=1}^{+\infty}g_{0,p}\kappa^{p-1}\frac{\Gamma(\kappa-p+1/2)}{\Gamma(\kappa-1/2)}\left(\frac{k_\perp^2\theta_{\alpha\perp}^2}{4\Omega_\alpha^2}\right)^p-\sum_\alpha\frac{2\omega_{p\alpha}^2}{k^2\theta_{\alpha\parallel}^2}\sum_{p=0}^{+\infty}g_{0,p}\frac{\kappa^p\Gamma(\kappa-p)}{\Gamma(\kappa)}\left(\frac{k_\perp^2\theta_{\alpha\perp}^2}{4\Omega_\alpha^2}\right)^p$$
$$\times\left\{\frac{k_\perp\theta_{\alpha\parallel}^2}{2\Omega_\alpha}\left(\frac{1}{L_{n\alpha}}-\frac{1}{L_{\theta\alpha\parallel}}+\frac{2p}{L_{\theta\alpha\perp}}\right)\frac{1}{k_\parallel\theta_{\alpha\parallel}}Z^{\mathrm{BL}}_{\kappa,p}(\varsigma^\kappa_\alpha)+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V'_{\alpha0}}{\Omega_\alpha}-\frac{k_\perp\theta_{\alpha\parallel}^2}{\Omega_\alpha L_{\theta\alpha\parallel}}\frac{\bar{\omega}_\alpha}{k_\parallel^2\theta_{\alpha\parallel}^2}\right)\frac{d}{d\varsigma^\kappa_\alpha}Z^{\mathrm{BL}}_{\kappa,p}(\varsigma^\kappa_\alpha)\right\}=0 \qquad (46)$$

or, in terms of b_α, $\eta_{\alpha\parallel}$, $\eta_{\alpha\perp}$, and $\omega_{*\alpha}$, is

$$1-\sum_\alpha\frac{1}{k^2\lambda_{D\alpha\perp}^2}\frac{2\kappa}{2\kappa-3}\sum_{p=1}^{+\infty}g_{0,p}\frac{\kappa^{p-1}\Gamma(\kappa-p+1/2)}{\Gamma(\kappa-1/2)}\left(\frac{2\kappa-3}{4\kappa}b_\alpha\right)^p-\sum_\alpha\frac{1}{k^2\lambda_{D\alpha\parallel}^2}\frac{2\kappa}{2\kappa-3}\sum_{p=0}^{+\infty}g_{0,p}\frac{\kappa^p\Gamma(\kappa-p)}{\Gamma(\kappa)}\left(\frac{2\kappa-3}{4\kappa}b_\alpha\right)^p$$
$$\times\left\{\left(\frac{2\kappa-3}{2\kappa}\right)^{1/2}\left(1-\frac{1}{2}\eta_{\alpha\parallel}+p\eta_{\alpha\perp}\right)\frac{\omega_{*\alpha}}{\sqrt{2}k_\parallel V_{T\alpha\parallel}}Z^{\mathrm{BL}}_{\kappa,p}(\varsigma^\kappa_\alpha)+\frac{1}{2}\left(1-\frac{k_\perp}{k_\parallel}\frac{V'_{\alpha0}}{\Omega_\alpha}-\eta_{\alpha\parallel}\frac{\omega_{*\alpha}\bar{\omega}_\alpha}{2k_\parallel^2V_{T\alpha\parallel}^2}\right)\frac{d}{d\varsigma^\kappa_\alpha}Z^{\mathrm{BL}}_{\kappa,p}(\varsigma^\kappa_\alpha)\right\}=0. \qquad (47)$$

Here $\varsigma^\kappa_\alpha=[2\kappa/(2\kappa-3)]^{1/2}\bar{\omega}_\alpha/(\sqrt{2}k_\parallel V_{T\alpha\parallel})$. It can be verified by following the analysis given for the product bi-Lorentzian that Eq. (45) → Eq. (24) and Eq. (47) → Eq. (26) in the limit as $\kappa\to\infty$.

IV. LOW FREQUENCY AND LONG PERPENDICULAR WAVELENGTH ($b_\alpha \ll 1$) MODES

In this section, we present analysis of the dispersion relations for the low frequency waves, in order to illustrate the differences between the equilibrium velocity distributions. For analytical tractability we restrict ourselves to the situation where $|\bar{\omega}_e/(\sqrt{2}k_\parallel V_{Te\parallel})|\ll 1$ and $|\bar{\omega}_i/(\sqrt{2}k_\parallel V_{Ti\parallel})|\gg 1$. Furthermore, we consider long perpendicular wavelength modes such that $b_e\approx 0$ and $b_i\ll 1$. These are the limiting conditions under which drift waves, ion-acoustic modes, and ion temperature gradient driven modes are excited. We start this section with the review of the previously known results for the bi-Maxwellian case and then discuss the various kappa-distribution cases.

(a) Bi-Maxwellian (BM): We refer to Eq. (26). Using $\Gamma_0(b_e)\approx 1$, $\Gamma_0(b_i)\cong 1-b_i$, $k\lambda_{De\parallel}\ll 1$ (quasineutrality condition), and the expansions[18]

$$Z^{\mathrm{BM}}(\xi_e)=i\sqrt{\pi}(1-\xi_e^2+\cdots)-2\xi_e\left(1-\frac{2}{3}\xi_e^2+\cdots\right),$$
$$|\xi_e|\ll 1, \qquad (48)$$

$$Z^{\mathrm{BM}}(\xi_i)=i\sqrt{\pi}\exp(-\xi_i^2)-\frac{1}{\xi_i}\left(1+\frac{1}{2\xi_i^2}+\cdots\right),\quad |\xi_i|\gg 1 \qquad (49)$$

in Eq. (26) and then keeping the leading terms, we find

$$1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}+k_\perp^2\rho_s^2-\frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})]$$
$$-\left[1-\frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i}-\frac{\omega_{*i}}{\bar{\omega}_i}(1+\eta_{i\parallel}-b_i\eta_{i\perp})\right]\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}$$
$$+i\sqrt{\frac{\pi}{2}}\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}-\frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]$$
$$+i\sqrt{\frac{\pi}{2}}\left(\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^3\left\{\left[1-\frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i}-\frac{\omega_{*i}}{\bar{\omega}_i}\left(1-\frac{1}{2}\eta_{i\parallel}\right.\right.\right.$$
$$\left.\left.\left.-b_i\eta_{i\perp}\right)\right]\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}+\frac{1}{2}\eta_{i\parallel}\frac{\omega_{*e}}{\bar{\omega}_i}\right\}\exp\left(-\frac{\bar{\omega}_i^2}{2k_\parallel^2V_{Ti\parallel}^2}\right)=0, \qquad (50)$$

where $\rho_s^2=c_s^2/\Omega_i^2$, $c_s^2=T_{e\parallel}/m_i$, and we have used $\omega_{*e}=-(T_{e\parallel}/T_{i\parallel})\omega_{*i}$.

We first consider the drift waves ($\bar{\omega}_i\sim\omega_{*e}$), which are realized when $k_\parallel$ is so small that $k_\parallel c_s\ll\omega_{*e}$. Drift waves have been discussed extensively in the plasma-physics literature.[19] Here we discuss some of the salient properties. The relevant dispersion relation that follows from Eq. (50) is

$$1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}+k_\perp^2\rho_s^2-[1-b_i(1+\eta_{i\perp})]\frac{\omega_{*e}}{\bar{\omega}_i}$$
$$+i\sqrt{\frac{\pi}{2}}\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}-\frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]$$
$$+i\sqrt{\frac{\pi}{8}}\eta_{i\parallel}\frac{\omega_{*e}\bar{\omega}_i^2}{k_\parallel^3V_{Ti\parallel}^3}\exp\left(-\frac{\bar{\omega}_i^2}{2k_\parallel^2V_{Ti\parallel}^2}\right)=0 \tag{51}$$

and its approximate solution is

$$\mathrm{Re}\,\bar{\omega}_i\cong A(k_\perp,k_\parallel)[1-b_i(1+\eta_{i\perp})]\omega_{*e}, \tag{52}$$

$$\mathrm{Im}\,\bar{\omega}_i\cong\sqrt{\frac{\pi}{2}}A(k_\perp,k_\parallel)\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Te\parallel}}\left\{\left[\left(1-\frac{1}{2}\eta_{e\parallel}\right)\omega_{*e}-\left(1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}\right)\mathrm{Re}\,\bar{\omega}_e\right]-\eta_{i\parallel}\omega_{*e}\frac{V_{Te\parallel}}{V_{Ti\parallel}}\left(\frac{\mathrm{Re}\,\bar{\omega}_i}{\sqrt{2}k_\parallel V_{Ti\parallel}}\right)^2\times\exp\left[-\left(\frac{\mathrm{Re}\,\bar{\omega}_i}{\sqrt{2}k_\parallel V_{Ti\parallel}}\right)^2\right]\right\}, \tag{53}$$

where

$$A(k_\perp,k_\parallel)=\frac{1}{1+(k_\perp/k_\parallel)(V_{e0}'/\Omega_e)+k_\perp^2\rho_s^2}. \tag{54}$$

We recall here that $\bar{\omega}_i=\omega-k_\parallel V_{i0}$ and $\bar{\omega}_e=\omega-k_\parallel V_{e0}=\bar{\omega}_i-k_\parallel V_d$, where $V_d=V_{e0}-V_{i0}$ is the drift speed of the electrons relative to the ions. The solution describes drift waves (in a reference frame moving with the ion flow velocity) modified by the presence of temperature gradients ($\eta_{e\parallel},\eta_{i\perp}$) and sheared electron flow velocity (V_{e0}'). Typically, $|(k_\perp/k_\parallel)(V_{e0}'/\Omega_e)|$ is very small compared to unity and may be neglected. The first term in Eq. (53) represents resonant interaction of electrons with the waves (Landau resonance) and it gives rise to instability when the proper condition is met. The last term in Eq. (53) represents resonant interaction of ions with the waves and leads to Landau damping or growth depending on the sign of $\eta_{i\parallel}$. However, this term is small for $\bar{\omega}_i\sim\omega_{*e}\gg k_\parallel c_s$ and $T_{e\parallel}\gtrsim T_{i\parallel}$. Equations (52)–(54) indicate that both $\eta_{i\perp}>0$ and $V_d>0$ favor the excitation of the drift waves, whereas, $\eta_{e\parallel}>0$ and $\eta_{i\parallel}>0$ oppose the excitation. For instance, if the small quantities, b_i, $|(k_\perp/k_\parallel)(V_{e0}'/\Omega_e)|$, and the ion Landau resonance term, are neglected in Eqs. (52)–(54), then we have

$$\mathrm{Re}\,\bar{\omega}_i\cong\frac{\omega_{*e}}{1+k_\perp^2\rho_s^2}, \tag{55}$$

$$\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\bar{\omega}_i}\cong\sqrt{\frac{\pi}{2}}\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Te\parallel}}\left(\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2}-\frac{1}{2}\eta_{e\parallel}+\frac{k_\parallel V_d}{\omega_{*e}}\right). \tag{56}$$

Solution (56) indicates instability ($\mathrm{Im}\,\omega>0$) for $V_d>0$ and $\eta_{e\parallel}<0$. When $\eta_{e\parallel}>0$ and the first term within the parentheses of Eq. (56) can be neglected in comparison to $\eta_{e\parallel}/2$, the instability condition ($\mathrm{Im}\,\omega>0$) for the current-driven drift wave is

$$V_d>\frac{\omega_{*e}}{2k_\parallel}\eta_{e\parallel}. \tag{57}$$

On the other hand, for $V_d=0$ (no current) and $\eta_{e\parallel}>0$, the instability condition ($\mathrm{Im}\,\omega>0$) is

$$\frac{1}{2}\eta_{e\parallel}<\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2}. \tag{58}$$

If $\eta_{e\parallel}=0$, the instability condition is satisfied with any $k_\perp^2\rho_s^2>0$. This is a well-known result and the instability is often referred to as the "universal" instability.[19] Next, we consider $\bar{\omega}_i\sim k_\parallel c_s\gg\omega_{*e}$. Assuming further that $k_\perp^2\rho_s^2\ll 1$, the dispersion relation (50) simplifies into

$$1-\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}\Lambda^2+i\sqrt{\frac{\pi}{2}}\left[\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}+\Lambda^2\frac{T_{e\parallel}}{T_{i\parallel}}\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\times\exp\left(-\frac{\bar{\omega}_i^2}{2k_\parallel^2V_{Ti\parallel}^2}\right)\right]=0, \tag{59}$$

where Λ is the velocity shear parameter defined by

$$\Lambda^2=\frac{1-(k_\perp/k_\parallel)(V_{i0}'/\Omega_i)}{1+(k_\perp/k_\parallel)(V_{e0}'/\Omega_e)}. \tag{60}$$

If $\Lambda^2>0$ so that Λ is real, the approximate solution of Eq. (59) is

$$\frac{\bar{\omega}_i}{k_\parallel c_s}\cong\Lambda\left\{1+i\sqrt{\frac{\pi}{8}}\left[\sqrt{\frac{m_e}{m_i}}\left(\frac{V_d}{c_s}-\Lambda\right)-\left(\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda^2\right)^{3/2}\times\exp\left(-\frac{T_{e\parallel}}{2T_{i\parallel}}\Lambda^2\right)\right]\right\}. \tag{61}$$

It describes current-driven ion-acoustic waves (in the reference frame moving with the ion flow velocity) in the presence of velocity shear, which becomes unstable ($\mathrm{Im}\,\omega>0$) when V_d (i.e., current) exceeds a threshold value V_d^0, where

$$\frac{V_d^0}{c_s}=\Lambda+\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda^2\right)^{3/2}\exp\left(-\frac{T_{e\parallel}}{2T_{i\parallel}}\Lambda^2\right). \tag{62}$$

The second term on the right-hand side, which represents ion Landau damping, can be made smaller by increasing the value of $T_{e\parallel}/T_{i\parallel}$ and Λ, so that the instability can be excited with a lower value of V_d^0. In the absence of velocity shear (i.e., $\Lambda^2=1$) and for electron-proton plasma, Eq. (62) yields $V_d^0/c_s\cong$ 10.13, 2.38, 1.17, and 1.0 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively. In the presence of velocity shear (i.e., $\Lambda^2\neq 1$), minimizing Eq. (62) with respect to Λ we find that the minimum value of V_d^0/c_s is obtained for $\Lambda=\Lambda_m$, where Λ_m is to be determined from

$$1+\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{3/2}\Lambda_m^2\left(3-\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_m^2\right)\exp\left(-\frac{T_{e\parallel}}{2T_{i\parallel}}\Lambda_m^2\right)=0. \tag{63}$$

For electron-ion plasma, Eq. (63) yields $\Lambda_m \cong 1.478$, 1.221, 1.066, and 0.88 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively, i.e., Λ_m increases as $T_{e\parallel}/T_{i\parallel}$ decreases. Consequently, ion Landau damping for smaller values of $T_{e\parallel}/T_{i\parallel}$ is reduced by the velocity shear. Substituting these values in Eq. (62) we find the minimum values of $V_d^0/c_s \cong 1.56$, 1.28, 1.12, and 0.92 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively. Since $|(k_\perp/k_\parallel)(V'_{e0}/\Omega_e)|$ is typically negligible, Λ can be made larger than unity for $|V'_{i0}/\Omega_i|<1$ by adjusting the values of $k_\perp/k_\parallel$. Comparison with the homogeneous (no velocity shear) case indicates that ion velocity shear allows excitation of ion acoustic instability with significantly smaller currents for lower values of $T_{e\parallel}/T_{i\parallel}$, and that the threshold current does not have as strong a dependence on $T_{e\parallel}/T_{i\parallel}$ as it is in the homogeneous case. This benign effect of velocity shear was first pointed out by Gavrishchaka *et al.*,[20] who showed by numerical solution of the dispersion relation (26) (without the density and temperature gradients) that the threshold current for ion-acoustic instability is significantly smaller in the presence of shear and that the threshold value is almost insensitive to variations of $T_{e\parallel}/T_{i\parallel}$ over a wide range of values of $T_{e\parallel}/T_{i\parallel}$ (from 0.1 to 10).

If $\Lambda^2<0$ so that Λ is pure imaginary, and if the small imaginary terms are neglected, Eq. (59) describes a purely growing mode with

$$\bar{\omega}_i = i|\Lambda|k_\parallel c_s. \tag{64}$$

This is referred to as the shear-driven ion-acoustic instability.[21]

Another instability that has been discussed in the literature is the ion temperature gradient driven instability.[22–24] This instability, like the shear-driven instability, does not rely on or is not affected by Landau resonance (wave-particle resonance). It is realized when $\eta_{i\parallel}\gg 1$ and in the limits when (50) is reduced to

$$1+\frac{k_\parallel^2 c_s^2}{\bar{\omega}_i^3}\omega_{*i}\eta_{i\parallel}=0. \tag{65}$$

It has the solution

$$\bar{\omega}_i=\frac{1}{2}(1\pm i\sqrt{3})\left[\frac{k_\parallel^2 c_s^2}{(\omega_{*i}\eta_{i\parallel})^2}\right]^{1/3}\omega_{*i}\eta_{i\parallel}, \tag{66}$$

where the upper sign (+) is for $\omega_{*i}\eta_{i\parallel}>0$ and the lower sign (−) is for $\omega_{*i}\eta_{i\parallel}<0$.

In the remainder of this section, we examine the same low frequency waves when different kappa distributions are used to describe the equilibrium plasma state.

(b) Kappa-Maxwellian: The series and asymptotic expansions of $Z_\kappa^{\mathrm{KM}}(\varsigma)$, which appears in Eq. (31) and which is the same as $Z_{\kappa M}(\varsigma)$ introduced by Hellberg and Mace,[9] are

$$Z_\kappa^{\mathrm{KM}}(\varsigma_e)=\frac{i\sqrt{\pi}\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}(1-\varsigma_e^2+\cdots)-\frac{2\kappa-1}{\kappa}\varsigma_e\left(1-\frac{2\kappa+1}{3\kappa}\varsigma_e^2+\cdots\right),\quad |\varsigma_e|\ll 1, \tag{67}$$

$$Z_\kappa^{\mathrm{KM}}(\varsigma_i)=\frac{i\sqrt{\pi}\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\frac{1}{(1+\varsigma_i^2/\kappa)^\kappa}-\frac{1}{\varsigma_i}\left(1+\frac{\kappa}{2\kappa-3}\frac{1}{\varsigma_i^2}+\cdots\right),\quad |\varsigma_i|\gg 1, \tag{68}$$

for integer values of κ. For noninteger (excluding half-integers) values of κ, the series expansion (67) remains unchanged; but the first term in the asymptotic expansion (68) is modified[9] as $i\to i-\tan\kappa\pi$. For half-integer values of κ, one can first relate Z_κ^{KM} to Z_κ^* of Summers and Thorne[1] [see Eq. (60) of Ref. 9] and then use the series and asymptotic expansions of Z_κ^* for half-integer values of κ given by Summers *et al.*[25] The result is that the series expansion Eq. (67) remains unchanged; but the first term in the asymptotic expansion Eq. (68) is modified as $i\pi\to i\pi+\log(a/\varsigma_i^2)$, where a is some number. Thus, noninteger (including half-integer) values of κ will add very small corrections to the real part of the frequencies of the waves considered here. We neglect the modifications and use Eqs. (67) and (68) in the following analysis. Using $\Gamma_0(b_e)\approx 1$, $\Gamma_0(b_i)\cong 1-b_i$, $k\lambda_{De\parallel}\ll 1$ (quasineutrality condition), and the expansions (67) and (68) in Eq. (31), and then keeping the leading terms, we find

$$\begin{aligned}&\frac{2\kappa-1}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)+k_\perp^2\rho_s^2-\frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})]-\left[1-\frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i}-\frac{\omega_{*i}}{\bar{\omega}_i}(1+\eta_{i\parallel}-b_i\eta_{i\perp})\right]\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}\\&+i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[\frac{2\kappa}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)-\frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]+i\sqrt{\frac{\pi}{2}}F(\kappa)\left(\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^3\\&\times\left\{\left[\frac{2\kappa}{2\kappa-3}\left(1-\frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i}\right)-\frac{\omega_{*i}}{\bar{\omega}_i}\left(1-\frac{1}{2}\eta_{i\parallel}-b_i\eta_{i\perp}\right)\right]\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}+\frac{1}{2\kappa-3}\frac{\omega_{*e}}{\bar{\omega}_i}\left[1+\left(\kappa-\frac{1}{2}\right)\eta_{i\parallel}-b_i\eta_{i\perp}\right]\right\}\\&\times\left(1+\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa+1)}=0,\end{aligned} \tag{69}$$

where

$$F(\kappa)=\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\left(\frac{2\kappa}{2\kappa-3}\right)^{1/2}. \qquad (70)$$

The other notations are the same as before. It may be verified that $F(\kappa)\to 1$ and Eq. (69) $\to$ Eq. (50) as $\kappa\to\infty$.

In the drift wave approximation ($\bar{\omega}_i\sim\omega_{*e}\gg k_\parallel c_s$), the dispersion relation becomes simplified as

$$\begin{aligned}&\frac{2\kappa-1}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)+k_\perp^2\rho_s^2-\frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})]\\&+i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[\frac{2\kappa}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)\right.\\&\left.-\frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]+i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\omega_{*e}}{k_\parallel V_{Ti\parallel}}\\&\times\left[1+\left(\kappa-\frac{1}{2}\right)\eta_{i\parallel}-b_i\eta_{i\perp}\right]\left(\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)\\&\times\left(1+\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa+1)}=0 \qquad (71)\end{aligned}$$

and its approximate solution is

$$\mathrm{Re}\,\bar{\omega}_i\cong B(k_\perp,k_\parallel)[1-b_i(1+\eta_{i\perp})]\omega_{*e}, \qquad (72)$$

$$\begin{aligned}\mathrm{Im}\,\omega\cong&\sqrt{\frac{\pi}{2}}B(k_\perp,k_\parallel)F(\kappa)\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Te\parallel}}\left\{\left(1-\frac{1}{2}\eta_{e\parallel}\right)\omega_{*e}\right.\\&-\frac{2\kappa}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)\mathrm{Re}\,\bar{\omega}_e-\omega_{*e}\frac{V_{Te\parallel}}{V_{Ti\parallel}}\\&\times\left[1+\left(\kappa-\frac{1}{2}\right)\eta_{i\parallel}-b_i\eta_{i\perp}\right]\left[\frac{1}{2\kappa-3}\left(\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^2\right]\\&\left.\times\left[1+\frac{1}{2\kappa-3}\left(\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^2\right]^{-(\kappa+1)}\right\}, \qquad (73)\end{aligned}$$

$$B(k_\perp,k_\parallel)=\frac{1}{[(2\kappa-1)/(2\kappa-3)][1+(k_\perp/k_\parallel)(V'_{e0}/\Omega_e)]+k_\perp^2\rho_s^2}. \qquad (74)$$

Both the real and the imaginary parts of ω are modified from their values for the bi-Maxwellian distribution. The ion Landau damping term in $\mathrm{Im}\,\omega$ (third term within the curly bracket) has a power law, instead of exponential dependence on $\mathrm{Re}\,\bar{\omega}_i/k_\parallel V_{Ti\parallel}$ when $\mathrm{Re}\,\bar{\omega}_i/k_\parallel V_{Ti\parallel}\gg 1$ and, hence, is larger than that for the bi-Maxwellian distribution in the parameter regime $\mathrm{Re}\,\bar{\omega}_i/k_\parallel V_{Ti\parallel}\gg 1$. For the limiting conditions that led to Eqs. (55) and (56) we now find

$$\mathrm{Re}\,\bar{\omega}_i\cong\frac{\omega_{*e}}{(2\kappa-1)/(2\kappa-3)+k_\perp^2\rho_s^2}, \qquad (75)$$

$$\begin{aligned}\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\bar{\omega}_i}\cong&\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Te\parallel}}\left[\frac{k_\perp^2\rho_s^2-1/(2\kappa-3)}{(2\kappa-1)/(2\kappa-3)+k_\perp^2\rho_s^2}\right.\\&\left.-\frac{1}{2}\eta_{e\parallel}+\frac{2\kappa}{2\kappa-3}\frac{k_\parallel V_d}{\omega_{*e}}\right]. \qquad (76)\end{aligned}$$

A comparison of Eq. (75) with Eq. (55) shows that $\mathrm{Re}\,\bar{\omega}_i$ is reduced from its value for the bi-Maxwellian distribution, as $(2\kappa-1)/(2\kappa-3)>1$. When $\eta_{e\parallel}>0$ and the first term within the parentheses of Eq. (76) can be neglected in comparison to $\eta_{e\parallel}/2$, the instability condition ($\mathrm{Im}\,\omega>0$) for the current-driven drift wave is

$$V_d>\left(\frac{2\kappa-3}{2\kappa}\right)\frac{\omega_{*e}}{2k_\parallel}\eta_{e\parallel}. \qquad (77)$$

A comparison with Eq. (57) shows that the threshold current for the instability is reduced from its value for the bi-Maxwellian distribution by the factor $(2\kappa-3)/2\kappa$. In the absence of current ($V_d=0$) and for $\eta_{e\parallel}>0$, the instability condition is

$$\frac{1}{2}\eta_{e\parallel}<\frac{k_\perp^2\rho_s^2-1/(2\kappa-3)}{(2\kappa-1)/(2\kappa-3)+k_\perp^2\rho_s^2}. \qquad (78)$$

If $\eta_{e\parallel}=0$, the instability condition is satisfied for $k_\perp^2\rho_s^2>1/(2\kappa-3)$, which is more stringent than the condition $k_\perp^2\rho_s^2>0$ obtained for the bi-Maxwellian distribution.

Next, we consider the ion-acoustic waves for which $\bar{\omega}_i\sim k_\parallel c_s\gg\omega_{*e}$. Assuming further that $k_\perp^2\rho_s^2\ll 1$, the dispersion relation that follows from Eq. (69) is

$$\begin{aligned}&\frac{2\kappa-1}{2\kappa-3}-\Lambda^2\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}+i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{2\kappa}{2\kappa-3}\\&\times\left[\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}+\Lambda^2\frac{T_{e\parallel}}{T_{i\parallel}}\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\left(1+\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa+1)}\right]\\&=0 \qquad (79)\end{aligned}$$

and its approximate solution for $\Lambda^2>0$ is

$$\mathrm{Re}\,\bar{\omega}_i\cong\Lambda_\kappa k_\parallel c_s, \qquad (80)$$

$$\begin{aligned}\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\bar{\omega}_i}\cong&\sqrt{\frac{\pi}{8}}F(\kappa)\frac{2\kappa}{2\kappa-1}\left[\sqrt{\frac{m_e}{m_i}}\left(\frac{V_d}{c_s}-\Lambda_\kappa\right)\right.\\&\left.-\frac{2\kappa-1}{2\kappa-3}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_\kappa^2\right)^{3/2}\left(1+\frac{\Lambda_\kappa^2}{2\kappa-3}\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{-(\kappa+1)}\right], \qquad (81)\end{aligned}$$

where

$$\Lambda_\kappa^2=\left(\frac{2\kappa-3}{2\kappa-1}\right)\Lambda^2. \qquad (82)$$

We again note that both the real and the imaginary parts of ω are modified from their values for the bi-Maxwellian distri-

bution. In particular, the power-law, instead of exponential, dependence of the ion Landau damping term (second term within braces in Im ω) on $T_{e\parallel}/T_{i\parallel}$, when $T_{e\parallel}/T_{i\parallel}\gg 1$, results in larger ion Landau damping and changes the instability condition significantly. The threshold value V_d^0 for instability is now given by

$$\frac{V_d^0}{c_s}=\Lambda_\kappa+\frac{2\kappa-1}{2\kappa-3}\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_\kappa^2\right)^{3/2}\times\left(1+\frac{\Lambda_\kappa^2}{2\kappa-3}\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{-(\kappa+1)}. \quad (83)$$

In the absence of velocity shear ($\Lambda^2=1$), $\Lambda_\kappa^2=(2\kappa-3)/(2\kappa-1)<1$. If we take $\kappa=3$, for example, then $\Lambda_\kappa^2=3/5$. For electron-proton plasma, Eq. (83) yields $V_d^0/c_s\cong 13.73$, 8.31, 5.52, and 3.05 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively. For a smaller (larger) value of κ, the values of V_d^0/c_s are larger (smaller). A comparison with the corresponding results for the bi-Maxwellian distribution shows that the threshold currents are larger for the kappa-Maxwellian distribution due to the larger ion Landau damping rates.

In the presence of velocity shear ($\Lambda^2\neq 1$), minimizing Eq. (83) with respect to Λ_κ we find that the minimum value of V_d^0/c_s is obtained for $\Lambda_\kappa=\Lambda_{\kappa m}$, where $\Lambda_{\kappa m}$ is determined from

$$\left(1+\frac{\Lambda_{\kappa m}^2}{2\kappa-3}\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{\kappa+2}=\frac{2\kappa-1}{2\kappa-3}\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{3/2}\times\left(\frac{2\kappa-1}{2\kappa-3}\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_{\kappa m}^2-3\right)\Lambda_{\kappa m}^2. \quad (84)$$

Using Eq. (84) in Eq. (83) the minimum value of V_d^0/c_s is obtained as

$$\left(\frac{V_d^0}{c_s}\right)_{\min}=\frac{[2\kappa/(2\kappa-3)](T_{e\parallel}/T_{i\parallel})\Lambda_{\kappa m}^2-2}{[(2\kappa-1)/(2\kappa-3)](T_{e\parallel}/T_{i\parallel})\Lambda_{\kappa m}^2-3}\Lambda_{\kappa m}. \quad (85)$$

For electron-proton plasma and for $\kappa=3$, for example, Eq. (84) yields $\Lambda_{\kappa m}\cong 1.986$, 1.681, 1.496, and 1.269 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively, i.e., $\Lambda_{\kappa m}$ increases as $T_{e\parallel}/T_{i\parallel}$ decreases. Substituting these values into Eq. (85) we find the minimum values of $V_d^0/c_s\cong 2.43$, 2.06, 1.83, and 1.55 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively. For a smaller (larger) value of κ, the minimum values of V_d^0/c_s are larger (smaller). Thus, in the presence of velocity shear, the threshold currents for the ion-acoustic instability are reduced more significantly for smaller values of $T_{e\parallel}/T_{i\parallel}$, as in the case of the bi-Maxwellian distribution, but the reduced values are still larger than those for the bi-Maxwellian distribution. This again is due to the increased ion Landau damping rates resulting from the kappa-Maxwellian distribution.

The previously mentioned shear-driven ion-acoustic instability, which is excited when $\Lambda_\kappa^2<0$ (i.e., $\Lambda^2<0$), is now described by

$$\bar{\omega}_i=i|\Lambda_\kappa|k_\parallel c_s. \quad (86)$$

When compared with the bi-Maxwellian result [see Eq. (64)], the growth rate of the instability for kappa-Maxwellian distribution is reduced by the factor $[(2\kappa-3)/(2\kappa-1)]^{1/2}$ for the same value of $|\Lambda|$.

Referring to Eq. (69), the ion temperature gradient driven instability, mentioned above, is described by

$$\frac{2\kappa-1}{2\kappa-3}+\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^3}\omega_{*i}\eta_{i\parallel}=0 \quad (87)$$

with the solution

$$\bar{\omega}_i=\frac{1}{2}(1\pm i\sqrt{3})\left[\frac{2\kappa-3}{2\kappa-1}\frac{k_\parallel^2c_s^2}{(\omega_{*i}\eta_{i\parallel})^2}\right]^{1/3}\omega_{*i}\eta_{i\parallel}. \quad (88)$$

The upper sign (+) is for $\omega_{*i}\eta_{i\parallel}>0$ and the lower sign (−) is for $\omega_{*i}\eta_{i\parallel}<0$. When compared with the bi-Maxwellian result [see Eq. (66)], growth rate of the instability for kappa-Maxwellian distribution is reduced by the factor $[(2\kappa-3)/(2\kappa-1)]^{1/3}$, with other parameters remaining the same.

(c) Product Bi-Lorentzian (PBL): The dispersion function $Z_\kappa^{\rm PBL}(\varsigma)$, which appears in Eq. (38), is similar to $Z_\kappa^*(\varsigma)$ of Summers and Thorne.[1] The series and asymptotic expansions of $Z_\kappa^*(\varsigma)$ for integer and half-integer values of κ are given in Refs. 1 and 25, respectively. Mace and Hellberg[26] extended their results to noninteger (excluding half-integer) values of κ. The series expansion of $Z_\kappa^*(\varsigma)$ is same for all real values of κ, and, as explained above [Sec. IV (b)], the modification of the asymptotic expansion of $Z_\kappa^*(\varsigma)$ due to noninteger (including half-integer) values of κ may be neglected in our analysis. We, therefore, use the expansions for integer values of κ, given by

$$Z_\kappa^{\rm PBL}(\varsigma_e)=\frac{i\sqrt{\pi}\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)}\left(1-\frac{\kappa_\parallel+1}{\kappa_\parallel}\varsigma_e^2+\cdots\right)-\frac{2\kappa_\parallel+1}{\kappa_\parallel}\varsigma_e\left(1-\frac{2\kappa_\parallel+3}{3\kappa_\parallel}\varsigma_e^2+\cdots\right),\quad |\varsigma_e|\ll 1, \quad (89)$$

$$Z_\kappa^{\rm PBL}(\varsigma_i)=\frac{i\sqrt{\pi}\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)}\frac{1}{(1+\varsigma_i^2/\kappa_\parallel)^{\kappa_\parallel+1}}-\frac{1}{\varsigma_i}\left(1+\frac{\kappa_\parallel}{2\kappa_\parallel-1}\frac{1}{\varsigma_i^2}+\cdots\right),\quad |\varsigma_i|\gg 1. \quad (90)$$

Since $b_e\approx 0$, we keep only the $p=0$ term in the electron sum in Eq. (38), and in order to retain effects of the order of b_i we keep $p=0$ and $p=1$ terms for ions. Using $g_{0,0}=1$, $g_{0,1}=-2$ [see Eq. (34)], $k\lambda_{De\parallel}\ll 1$, and keeping the leading terms we find

$$\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)+k_\perp^2\rho_s^2-\frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})]-\left[1-\frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i}-\frac{\omega_{*i}}{\bar{\omega}_i}(1+\eta_{i\parallel}-b_i\eta_{i\perp})\right]\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}$$
$$+i\sqrt{\frac{\pi}{2}}F(\kappa_\parallel)\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[\frac{2\kappa_\parallel+2}{2\kappa_\parallel-1}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)-\frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]+i\sqrt{\frac{\pi}{2}}F(\kappa)\left(\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^3$$
$$\times\left\{\left[\frac{2\kappa_\parallel+2}{2\kappa_\parallel-1}\left(1-\frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i}\right)-\frac{\omega_{*i}}{\bar{\omega}_i}\left(1-\frac{1}{2}\eta_{i\parallel}-b_i\eta_{i\perp}\right)\right]\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}+\frac{1}{2\kappa_\parallel-1}\frac{\omega_{*e}}{\bar{\omega}_i}\left[1+\left(\kappa_\parallel+\frac{1}{2}\right)\eta_{i\parallel}-b_i\eta_{i\perp}\right]\right\}$$
$$\times\left(1+\frac{1}{2\kappa_\parallel-1}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa_\parallel+2)}=0. \tag{91}$$

Here,

$$F(\kappa_\parallel)=\frac{\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)}\left(\frac{2\kappa_\parallel}{2\kappa_\parallel-1}\right)^{1/2} \tag{92}$$

and the other notations are the same as before.

The simplified dispersion relation in the drift wave approximation ($\bar{\omega}_i\sim\omega_{*e}\gg k_\parallel c_s$) is

$$\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)+k_\perp^2\rho_s^2-\frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})]$$
$$+i\sqrt{\frac{\pi}{2}}F(\kappa_\parallel)\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[\frac{2\kappa_\parallel+2}{2\kappa_\parallel-1}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)\right.$$
$$\left.-\frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]+i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\omega_{*e}}{k_\parallel V_{Ti\parallel}}$$
$$\times\left[1+\left(\kappa_\parallel+\frac{1}{2}\right)\eta_{i\parallel}-b_i\eta_{i\perp}\right]\left(\frac{1}{2\kappa_\parallel-1}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)$$
$$\times\left(1+\frac{1}{2\kappa_\parallel-1}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa_\parallel+2)}=0 \tag{93}$$

and its approximate solution is

$$\mathrm{Re}\,\bar{\omega}_i\cong C(k_\perp,k_\parallel)[1-b_i(1+\eta_{i\perp})]\omega_{*e}, \tag{94}$$

$$\mathrm{Im}\,\omega\cong\sqrt{\frac{\pi}{2}}C(k_\perp,k_\parallel)F(\kappa_\parallel)\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Te\parallel}}\left\{\left(1-\frac{1}{2}\eta_{e\parallel}\right)\omega_{*e}\right.$$
$$-\frac{2\kappa_\parallel+2}{2\kappa_\parallel-1}\left(1+\frac{k_\perp}{k_\parallel}\frac{V'_{e0}}{\Omega_e}\right)\mathrm{Re}\,\bar{\omega}_e-\omega_{*e}\frac{V_{Te\parallel}}{V_{Ti\parallel}}$$
$$\times\left[1+\left(\kappa_\parallel+\frac{1}{2}\right)\eta_{i\parallel}-b_i\eta_{i\perp}\right]\left[\frac{1}{2\kappa_\parallel-1}\left(\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^2\right]$$
$$\left.\times\left[1+\frac{1}{2\kappa_\parallel-1}\left(\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^2\right]^{-(\kappa_\parallel+2)}\right\}, \tag{95}$$

$$C(k_\perp,k_\parallel)=\frac{1}{[(2\kappa_\parallel+1)/(2\kappa_\parallel-1)][1+(k_\perp/k_\parallel)(V'_{e0}/\Omega_e)]+k_\perp^2\rho_s^2}. \tag{96}$$

Both the real and the imaginary parts of ω are different from those for the bi-Maxwellian distribution as well as for the kappa-Maxwellian distribution. The power-law dependence of the ion Landau damping term (third term within the curly bracket in Im ω) on Re $\bar{\omega}_i/k_\parallel V_{Ti\parallel}$, when Re $\bar{\omega}_i/k_\parallel V_{Ti\parallel}\gg 1$, is also different from that for the kappa-Maxwellian distribution. This leads to an instability condition that is different not only from the condition for the bi-Maxwellian distribution but also from the condition for the kappa-Maxwellian distribution. As in the cases of bi-Maxwellian and kappa-Maxwellian distributions, we ignore $|(k_\perp/k_\parallel)(V'_{e0}/\Omega_e)|$ and the effects of finite ion temperature (i.e., we neglect b_i and ion Landau damping term). Then,

$$\mathrm{Re}\,\bar{\omega}_i\cong\frac{\omega_{*e}}{(2\kappa_\parallel+1)/(2\kappa_\parallel-1)+k_\perp^2\rho_s^2}, \tag{97}$$

$$\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\bar{\omega}_i}\cong\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\mathrm{Re}\,\bar{\omega}_i}{k_\parallel V_{Te\parallel}}\left[\frac{k_\perp^2\rho_s^2-1/(2\kappa_\parallel-1)}{(2\kappa_\parallel+1)/(2\kappa_\parallel-1)+k_\perp^2\rho_s^2}\right.$$
$$\left.-\frac{1}{2}\eta_{e\parallel}+\frac{2\kappa_\parallel+2}{2\kappa_\parallel-1}\frac{k_\parallel V_d}{\omega_{*e}}\right]. \tag{98}$$

A comparison of Eq. (97) with Eqs. (55) and (75) shows that Re $\bar{\omega}_i$ is reduced from its value for the bi-Maxwellian distribution, as $(2\kappa_\parallel+1)/(2\kappa_\parallel-1)>1$, and the reduced value is different from that for the kappa-Maxwellian distribution. When $\eta_{e\parallel}>0$ and the first term within the parentheses of Eq. (98) can be neglected in comparison to $\eta_{e\parallel}/2$, the instability condition (Im $\omega>0$) for the current-driven drift wave is

$$V_d>\left(\frac{2\kappa_\parallel-1}{2\kappa_\parallel+2}\right)\frac{\omega_{*e}}{2k_\parallel}\eta_{e\parallel}. \tag{99}$$

Thus the threshold current for instability is reduced by the factor $(2\kappa_\parallel-1)/(2\kappa_\parallel+2)$, when compared with that for the bi-Maxwellian distribution [see Eq. (57)]. Also, the reduction factor is different from that for the kappa-Maxwellian distri-

bution [see Eq. (77)]. In the absence of current ($V_d=0$) and for $\eta_{e\parallel}>0$, the instability condition is

$$\frac{1}{2}\eta_{e\parallel}<\frac{k_\perp^2\rho_s^2-1/(2\kappa_\parallel-1)}{(2\kappa_\parallel+1)/(2\kappa_\parallel-1)+k_\perp^2\rho_s^2}. \tag{100}$$

If $\eta_{e\parallel}=0$, the instability condition is satisfied for $k_\perp^2\rho_s^2>1/(2\kappa_\parallel-1)$, which is more stringent than the condition for the bi-Maxwellian distribution and is different from that for the kappa-Maxwellian distribution.

Next, we consider the ion-acoustic waves for which $\bar{\omega}_i\sim k_\parallel c_s\gg\omega_{*e}$. Assuming further that $k_\perp^2\rho_s^2\ll 1$, the dispersion relation that follows from Eq. (91) is

$$\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}-\Lambda^2\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}+i\sqrt{\frac{\pi}{2}}F(\kappa_\parallel)\frac{2\kappa_\parallel+2}{2\kappa_\parallel-1}\times\left[\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}+\Lambda^2\frac{T_{e\parallel}}{T_{i\parallel}}\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\times\left(1+\frac{1}{2\kappa_\parallel-1}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa_\parallel+2)}\right]=0 \tag{101}$$

and its approximate solution for $\Lambda^2>0$ is

$$\mathrm{Re}\,\bar{\omega}_i\cong\Lambda_{\kappa_\parallel}k_\parallel c_s, \tag{102}$$

$$\frac{\mathrm{Im}\,\bar{\omega}_i}{\mathrm{Re}\,\bar{\omega}_i}\cong\sqrt{\frac{\pi}{8}}F(\kappa_\parallel)\frac{2\kappa_\parallel+2}{2\kappa_\parallel+1}\left[\sqrt{\frac{m_e}{m_i}}\left(\frac{V_d}{c_s}-\Lambda_{\kappa_\parallel}\right)-\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_{\kappa_\parallel}^2\right)^{3/2}\left(1+\frac{\Lambda_{\kappa_\parallel}^2}{2\kappa_\parallel-1}\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{-(\kappa_\parallel+2)}\right], \tag{103}$$

where

$$\Lambda_{\kappa_\parallel}^2=\left(\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}\right)\Lambda^2. \tag{104}$$

Both the real and the imaginary parts of ω are different from their values for the bi-Maxwellian distribution as well as the kappa-Maxwellian distribution. The power-law dependence of the ion Landau damping term (second term within the braces in Im ω) on $T_{e\parallel}/T_{i\parallel}$, when $T_{e\parallel}/T_{i\parallel}\gg 1$, results in larger ion Landau damping rate than that for the bi-Maxwellian distribution, and it is different from the power-law dependence for the kappa-Maxwellian distribution. The threshold value V_d^0 for instability is

$$\frac{V_d^0}{c_s}=\Lambda_{\kappa_\parallel}+\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_{\kappa_\parallel}^2\right)^{3/2}\times\left(1+\frac{\Lambda_{\kappa_\parallel}^2}{2\kappa_\parallel-1}\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{-(\kappa_\parallel+2)}. \tag{105}$$

In the absence of velocity shear ($\Lambda^2=1$), $\Lambda_{\kappa_\parallel}^2=(2\kappa_\parallel-1)/(2\kappa_\parallel+1)$. If we take $\kappa_\parallel=3$, for example, then $\Lambda_{\kappa_\parallel}^2=5/7$. For electron-proton plasma, Eq. (105) yields $V_d^0/c_s\cong 14.4$, 7.7, 4.64, and 2.29 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively. For a smaller (larger) value of $\kappa_\parallel$, the values of V_d^0/c_s are larger (smaller).

In the presence of velocity shear ($\Lambda^2\neq 1$), minimizing Eq. (105) with respect to $\Lambda_{\kappa_\parallel}$ we find that the minimum value of V_d^0/c_s is obtained for $\Lambda_{\kappa_\parallel}=\Lambda_{\kappa_\parallel m}$, where $\Lambda_{\kappa_\parallel m}$ is determined from

$$\left(1+\frac{\Lambda_{\kappa_\parallel m}^2}{2\kappa_\parallel-1}\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{\kappa_\parallel+3}=\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{3/2}\times\left(\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\frac{T_{e\parallel}}{T_{i\parallel}}\Lambda_{\kappa_\parallel m}^2-3\right)\Lambda_{\kappa_\parallel m}^2. \tag{106}$$

With the use of Eq. (106) in Eq. (105) the minimum value of V_d^0/c_s is obtained as

$$\left(\frac{V_d^0}{c_s}\right)_{\min}=\frac{[(2\kappa_\parallel+2)/(2\kappa_\parallel-1)](T_{e\parallel}/T_{i\parallel})\Lambda_{\kappa_\parallel m}^2-2}{[(2\kappa_\parallel+1)/(2\kappa_\parallel-1)](T_{e\parallel}/T_{i\parallel})\Lambda_{\kappa_\parallel m}^2-3}\Lambda_{\kappa_\parallel m}. \tag{107}$$

For electron-proton plasma and for $\kappa_\parallel=3$, Eq. (106) yields $\Lambda_{\kappa_\parallel m}\cong 1.916$, 1.614, 1.429, and 1.203 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively, i.e., $\Lambda_{\kappa_\parallel m}$ increases as $T_{e\parallel}/T_{i\parallel}$ decreases. Substituting these values into Eq. (107) we find the minimum values of $V_d^0/c_s\cong 2.25$, 1.89, 1.67, and 1.40 for $T_{e\parallel}/T_{i\parallel}=10$, 15, 20, and 30, respectively. For a smaller (larger) value of $\kappa_\parallel$, the minimum values of V_d^0/c_s are larger (smaller).

A comparison of the results for the product bi-Lorentzian distribution with the corresponding results for the bi-Maxwellian and the kappa-Maxwellian distributions show that the threshold currents for ion-acoustic instability in the absence of velocity shear are larger than those for the bi-Maxwellian distribution, but are somewhat smaller than those for the kappa-Maxwellian distribution. In the presence of velocity shear, the threshold currents are more significantly reduced for smaller values of $T_{e\parallel}/T_{i\parallel}$, as in the cases of bi-Maxwellian and kappa-Maxwellian distributions. The reduced threshold currents are still larger than those for the bi-Maxwellian distribution, but are somewhat smaller than those for the kappa-Maxwellian distribution.

The shear-driven ion-acoustic instability, which is excited when $\Lambda_{\kappa_\parallel}^2<0$ (i.e., $\Lambda^2<0$), is now described by

$$\bar{\omega}_i=i|\Lambda_{\kappa_\parallel}|k_\parallel c_s. \tag{108}$$

When compared with the bi-Maxwellian result [see Eq. (64)], the growth rate of the instability is reduced by the factor $[(2\kappa_\parallel-1)/(2\kappa_\parallel+1)]^{1/2}$ for the same value of $|\Lambda|$. However, the growth rate is somewhat larger than that for the kappa-Maxwellian distribution [see Eq. (86)], as $|\Lambda_{\kappa_\parallel}|>|\Lambda_\kappa|$.

Referring to Eq. (91), the ion temperature gradient driven instability is now described by

$$\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}+\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^3}\omega_{*i}\eta_{i\parallel}=0 \tag{109}$$

with the solution

$$\bar{\omega}_i = \frac{1}{2}(1 \pm i\sqrt{3})\left[\frac{2\kappa_\parallel - 1}{2\kappa_\parallel + 1}\frac{k_\parallel^2 c_s^2}{(\omega_{*i}\eta_{i\parallel})^2}\right]^{1/3}\omega_{*i}\eta_{i\parallel}. \tag{110}$$

The upper sign (+) is for $\omega_{*i}\eta_{i\parallel}>0$ and the lower sign (−) is for $\omega_{*i}\eta_{i\parallel}<0$. The growth rate of the instability is reduced by the factor $[(2\kappa_\parallel-1)/(2\kappa_\parallel+1)]^{1/3}$, when compared with that for the bi-Maxwellian distribution [see Eq. (66)], and it is slightly larger than that for the kappa-Maxwellian distribution [see Eq. (88)], other parameters remaining the same.

(d) Bi-Lorentzian Plasma: Since $b_e \approx 0$, we keep only the $p=0$ term in the electron sum in Eq. (47). Consequently, we need to know the series expansion of $Z_{\kappa,0}^{\mathrm{BL}}(\varsigma_e)$ for $\varsigma_e \ll 1$. For the ions, we keep $p=0$ and $p=1$ terms in Eq. (47) in order to retain effects of order b_i. Hence, we need to know the asymptotic expansions of $Z_{\kappa,0}^{\mathrm{BL}}(\varsigma_i)$ and $Z_{\kappa,1}^{\mathrm{BL}}(\varsigma_i)$ for $\varsigma_i \gg 1$. As we mentioned earlier, $Z_{\kappa,0}^{\mathrm{BL}}$ is same as $Z_\kappa^{\mathrm{KM}}(\varsigma)$ and so its series and asymptotic expansions are given by Eqs. (67) and (68), respectively [see the discussion following Eqs. (67) and (68)]. The asymptotic expansion of $Z_{\kappa,1}^{\mathrm{BL}}(\varsigma_i)$can be derived from the asymptotic expansion of $Z_{\kappa,0}^{\mathrm{BL}}(\varsigma_i)$ by using the relation (44). We thus have

$$Z_{\kappa,0}^{\mathrm{BL}}(\varsigma_e) = \frac{i\sqrt{\pi}\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}(1-\varsigma_e^2+\cdots) - \frac{2\kappa-1}{\kappa}\varsigma_e \times\left(1-\frac{2\kappa+1}{3\kappa}\varsigma_e^2+\cdots\right), \quad |\varsigma_e| \ll 1, \tag{111}$$

$$Z_{\kappa,0}^{\mathrm{BL}}(\varsigma_i) = \frac{i\sqrt{\pi}\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\frac{1}{(1+\varsigma_i^2/\kappa)^\kappa} - \frac{1}{\varsigma_i}\left(1+\frac{\kappa}{2\kappa-3}\frac{1}{\varsigma_i^2}+\cdots\right), \quad |\varsigma_i| \gg 1, \tag{112}$$

$$Z_{\kappa,1}^{\mathrm{BL}}(\varsigma_i) = \frac{i\sqrt{\pi}\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\frac{1}{(1+\varsigma_i^2/\kappa)^{\kappa-1}} - \frac{2(\kappa-1)}{2\kappa-3}\frac{1}{\varsigma_i}\left(1+\frac{\kappa}{2\kappa-5}\frac{1}{\varsigma_i^2}+\cdots\right), \quad |\varsigma_i| \gg 1. \tag{113}$$

Using the expansions in Eq. (47), assuming $k\lambda_{De\parallel} \ll 1$, and keeping only the leading terms, as before, we find

$$\frac{2\kappa-1}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}\right) + k_\perp^2\rho_s^2 - \frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})] - \left[1-\frac{k_\perp}{k_\parallel}\frac{V_{i0}'}{\Omega_i} - \frac{\omega_{*i}}{\bar{\omega}_i}\left(1+\eta_{i\parallel}-\frac{2\kappa-1}{2\kappa-5}b_i\eta_{i\perp}\right)\right]\frac{k_\parallel^2 c_s^2}{\bar{\omega}_i^2}$$
$$+ i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[\frac{2\kappa}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}\right) - \frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right] + i\sqrt{\frac{\pi}{2}}F(\kappa)\left(\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\right)^3$$
$$\times\left\{\left[\frac{2\kappa}{2\kappa-3}\left(1-\frac{k_\perp}{k_\parallel}\frac{V_{i0}'}{\Omega_i}\right) - \frac{\omega_{*i}}{\bar{\omega}_i}\left(1-\frac{1}{2}\eta_{i\parallel}-\frac{2\kappa-3}{2\kappa-2}b_i\eta_{i\perp}\right)\right]\frac{k_\parallel^2 c_s^2}{\bar{\omega}_i^2} + \frac{1}{2\kappa-3}\frac{\omega_{*e}}{\bar{\omega}_i}\left[1+\left(\kappa-\frac{1}{2}\right)\eta_{i\parallel}\right.\right.$$
$$\left.\left.-\frac{1}{2\kappa-2}\frac{\bar{\omega}_i^2}{k_\parallel^2 V_{Ti\parallel}^2}b_i\eta_{i\perp}\right]\right\}\left(1+\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2 V_{Ti\parallel}^2}\right)^{-(\kappa+1)} = 0, \tag{114}$$

where

$$F(\kappa) = \frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\left(\frac{2\kappa}{2\kappa-3}\right)^{1/2}. \tag{115}$$

The simplified dispersion relation in the drift wave approximation ($\bar{\omega}_i \sim \omega_{*e} \gg k_\parallel c_s$) is

$$\frac{2\kappa-1}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}\right) + k_\perp^2\rho_s^2 - \frac{\omega_{*e}}{\bar{\omega}_i}[1-b_i(1+\eta_{i\perp})] + i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}\left[\frac{2\kappa}{2\kappa-3}\left(1+\frac{k_\perp}{k_\parallel}\frac{V_{e0}'}{\Omega_e}\right) - \frac{\omega_{*e}}{\bar{\omega}_e}\left(1-\frac{1}{2}\eta_{e\parallel}\right)\right]$$
$$+ i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{\omega_{*e}}{k_\parallel V_{Ti\parallel}}\left[1+\left(\kappa-\frac{1}{2}\right)\eta_{i\parallel} - \frac{1}{2\kappa-2}\frac{\bar{\omega}_i^2}{k_\parallel^2 V_{Ti\parallel}^2}b_i\eta_{i\perp}\right]\left(\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2 V_{Ti\parallel}^2}\right)\left(1+\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2 V_{Ti\parallel}^2}\right)^{-(\kappa+1)} = 0. \tag{116}$$

We notice that, except for the contribution proportional to $b_i\eta_{i\perp}$ in the ion Landau damping term, Eq. (116) is identical to the corresponding dispersion relation for the kappa-Maxwellian distribution [see Eq. (71)]. So, in the limit when $b_i\eta_{i\perp} \ll 1$, the stability properties of the drift waves for bi-Lorentzian distributions are same as those for the kappa-Maxwellian distribution [see Eqs. (72)–(78)].

For the ion-acoustic waves ($\bar{\omega}_i \sim k_\parallel c_s \gg \omega_{*e}$) we assume, as before, that $k_\perp^2\rho_s^2 \ll 1$, and the dispersion relation that follows is

$$\frac{2\kappa-1}{2\kappa-3}-\Lambda^2\frac{k_\parallel^2c_s^2}{\bar{\omega}_i^2}+i\sqrt{\frac{\pi}{2}}F(\kappa)\frac{2\kappa}{2\kappa-3}\left[\frac{\bar{\omega}_e}{k_\parallel V_{Te\parallel}}+\Lambda^2\frac{T_{e\parallel}}{T_{i\parallel}}\frac{\bar{\omega}_i}{k_\parallel V_{Ti\parallel}}\left(1+\frac{1}{2\kappa-3}\frac{\bar{\omega}_i^2}{k_\parallel^2V_{Ti\parallel}^2}\right)^{-(\kappa+1)}\right]=0 \quad (117)$$

which is identical to the dispersion relation (79) for the kappa-Maxwellian distribution. Hence, the analysis of the ion-acoustic wave dispersion relation for bi-Lorentzian distribution is same as that for the kappa-Maxwellian distribution [see Eqs. (80)–(85)]. We mention here that the ion-acoustic instability for Lorentzian distribution (i.e., $T_{\alpha\parallel}=T_{\alpha\perp}$) and in the absence of velocity shear has been numerically analyzed by Meng *et al.*[5] The analyses for the shear-driven instability and the ion temperature-gradient driven instability for bi-Lorentzian distribution also remain the same as those for the kappa-Maxwellian distribution [see Eqs. (86)–(88)].

V. SUMMARY AND DISCUSSIONS

We have presented the linear dispersion relations for electrostatic waves in spatially inhomogeneous, current-carrying anisotropic plasma, where the equilibrium particle velocity distributions are modeled by various Lorentzian (kappa) distribution functions and by the well-known bi-Maxwellian distribution. Spatial inhomogeneities, assumed to be weak, include density gradients, temperature gradients, and gradients (shear) in the parallel (to the ambient magnetic field) flow velocities associated with the current. In order to illustrate the distinguishing features of the kappa distributions, stability properties of the low frequency (lower than ion cyclotron frequency) and long perpendicular wavelength (longer than ion gyroradius) modes have been studied in detail, and the results have been contrasted with those for the bi-Maxwellian distribution. Specific attention has been given to the drift waves, the current-driven ion-acoustic waves in the presence of velocity shear, the velocity shear-driven ion-acoustic modes, and the ion temperature gradient driven modes.

The growth rates of the drift wave instability and the current-driven ion-acoustic instability, both of which rely on wave-particle interactions for their excitation, are reduced from their values for the bi-Maxwellian distribution due to larger ion Landau damping rates associated with the kappa distributions. For the same reason, the excitation conditions for these two instabilities are more stringent in the case of the kappa distributions. The dominant ion Landau damping rates are proportional to $\operatorname{Im} Z'$ (prime on Z denotes derivative with respect to its argument), which have power-law dependence on $\operatorname{Re}\bar{\omega}_i/(k_\parallel V_{Ti\parallel})\gg 1$ for the kappa distributions. This is in sharp contrast with the exponential dependence for the bi-Maxwellian distribution. As a result, the ion Landau damping rates of plasma waves that are excited in the $\operatorname{Re}\bar{\omega}_i/(k_\parallel V_{Ti\parallel})\gg 1$ regime are larger in kappa-distribution plasmas than in bi-Maxwellian plasma. Figure 2 shows the marked differences in the behavior of $\operatorname{Im} Z'$'s associated with the different velocity distributions, when $\operatorname{Re}\bar{\omega}_i/(k_\parallel V_{Ti\parallel})\gg 1$. A particularly important consequence of the enhanced ion Landau damping rates is that the threshold currents for the ion-acoustic instability in kappa-distribution plasmas are larger than those in the bi-Maxwellian plasma, even in the presence of shear in the parallel flow velocity. Relativistic effects associated with the suprathermal ions that participate in the considered instabilities have been neglected in the present nonrelativistic treatment under the assumption $(T_{i\parallel},T_{i\perp})\ll m_ic^2$. For a recent paper on the modeling of energetic particles by relativistic kappa distribution, see Ref. 27.

The stability characteristics of the other two instabilities (shear-driven ion-acoustic instability and ion temperature-gradient driven instability), which do not rely on wave-particle interactions and for which Landau damping/growth terms in the dispersion relation may be neglected, can be better understood in terms of $(\tilde{n}_{e1}/n_0)$ and $(\tilde{n}_{i1}/n_0)$, where $\tilde{n}_{e1}$ and $\tilde{n}_{i1}$ are the density perturbations, since the dispersion relations are obtained by demanding quasineutrality $(\tilde{n}_{e1}=\tilde{n}_{i1})$. $(\tilde{n}_{e1}/n_0)$ and $(\tilde{n}_{i1}/n_0)$ are also helpful in understanding the origin of the reduced frequencies of the drift waves and the ion-acoustic waves in kappa-distribution plasmas. Under the conditions assumed in Sec. IV, the electron density perturbations for the different equilibrium distributions are

$$(\tilde{n}_{e1}/n_0)=\alpha(k_\perp,k_\parallel)\times\begin{cases} e\tilde{\phi}_1/T_{e\parallel}, & \text{for BM}\\ [(2\kappa-1)/(2\kappa-3)](e\tilde{\phi}_1/T_{e\parallel}), & \text{for KM}\\ [(2\kappa_\parallel+1)/(2\kappa_\parallel-1)](e\tilde{\phi}_1/T_{e\parallel}), & \text{for PBL}\\ \text{(Same as the expression for KM), for BL,} \end{cases} \quad (118)$$

where $\alpha(k_\perp,k_\parallel)=1+(k_\perp/k_\parallel)(V'_{e0}/\Omega_e)$. The differences in the expressions can be traced to the differences in the series expansions of Z for $\bar{\omega}_e/(k_\parallel V_{Te\parallel})\ll 1$ and to the differences in the relations of $\theta_{e\parallel}^2$ to $T_{e\parallel}$. The ion density perturbation, on the

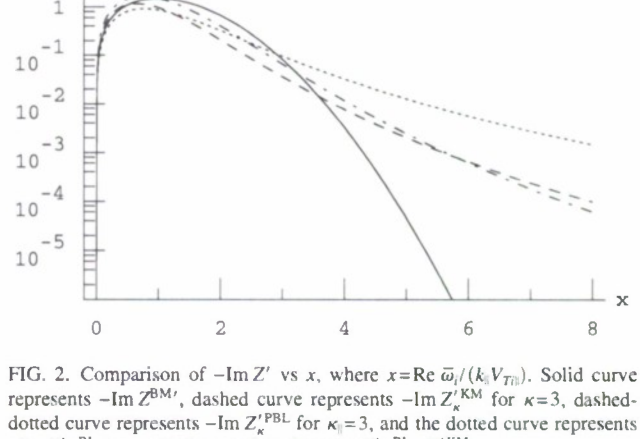

FIG. 2. Comparison of $-\operatorname{Im} Z'$ vs x, where $x=\operatorname{Re}\bar{\omega}_i/(k_\parallel V_{Ti\parallel})$. Solid curve represents $-\operatorname{Im} Z^{BM\prime}$, dashed curve represents $-\operatorname{Im} Z_\kappa'^{KM}$ for $\kappa=3$, dashed-dotted curve represents $-\operatorname{Im} Z_\kappa'^{PBL}$ for $\kappa_\parallel=3$, and the dotted curve represents $-\operatorname{Im} Z_{\kappa,1}'^{BL}$ for $\kappa=3$. As noted in the text, $Z_{\kappa,0}'^{BL}=Z_\kappa'^{KM}$.

other hand, is same for all the distributions and is given by

$$\left(\frac{\tilde{n}_{i1}}{n_0}\right) = \left\{\left[1 - \frac{k_\perp}{k_\parallel}\frac{V'_{i0}}{\Omega_i} - \frac{\omega_{*i}}{\bar{\omega}_i}(1+\eta_{i\parallel})\right]\frac{k_\parallel^2 V_{Ti\parallel}^2}{\bar{\omega}_i^2} - \frac{\omega_{*i}}{\bar{\omega}_i}\right\}\frac{e\tilde{\phi}_1}{T_{i\parallel}} \qquad (119)$$

when b_i and $k_\perp^2\rho_s^2$ are neglected for simplicity. This is due to the fact that the leading term in the asymptotic expansions of Z for $\bar{\omega}_i/(k_\parallel V_{Ti\parallel}) \gg 1$ is the same for all the distributions. Equation (118) shows that the adiabatic response of the electrons to the electrostatic potential perturbation in kappa-distribution plasmas is reduced from its value in bi-Maxwellian plasma. The reduced adiabatic response of the electrons explains the reduction of the growth rates of the shear-driven ion-acoustic instability and the ion temperature-gradient driven instability as well as the reduction of the frequencies of the drift waves and the ion-acoustic waves in kappa-distribution plasmas.

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